# TQS 211

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and perhaps have a different layout.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. The following is a graph recording the velocity of a car, $v(x)$ (in ten's of miles per hour) as a function of 10 minute intervals, x.
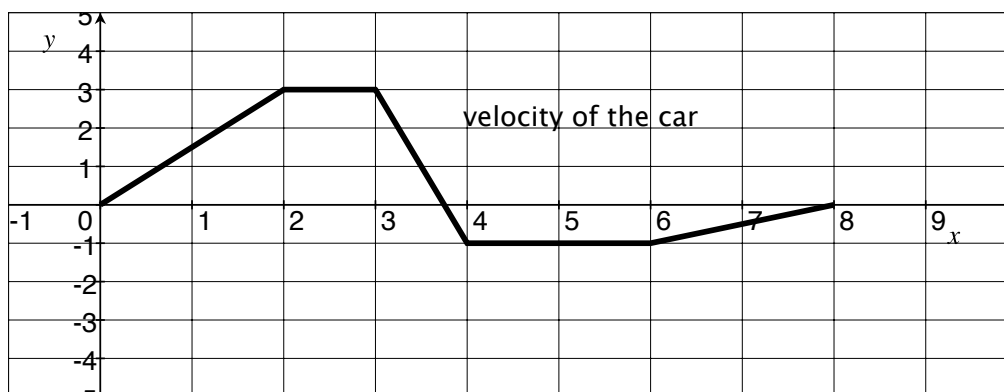
(a) Explain what $v'(x)$ is in physical terms. Consider explaining specific examples like $v'(1)$ or $v'(3.5)$.

(b) Explain what $\int_0^t V(x)\,dx$ is in physical terms. Consider explaining specific examples like when t is 3 or when t is 5.

2. (§3.4 #36) Find the equation of the tangent line to $f(x) = \frac{2x-5}{x+1}$ at the point at which $x = 0$.

3. Let f be the function whose graph is on the right and g be the function whose graph is on the left.
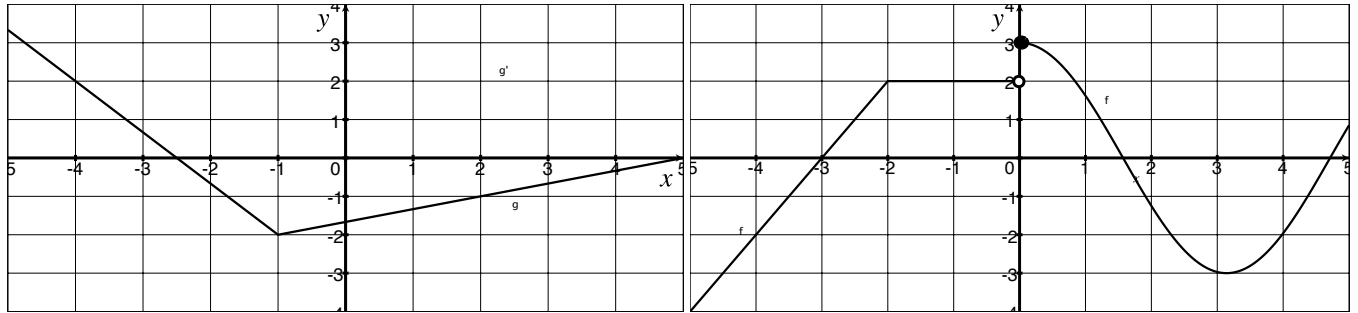
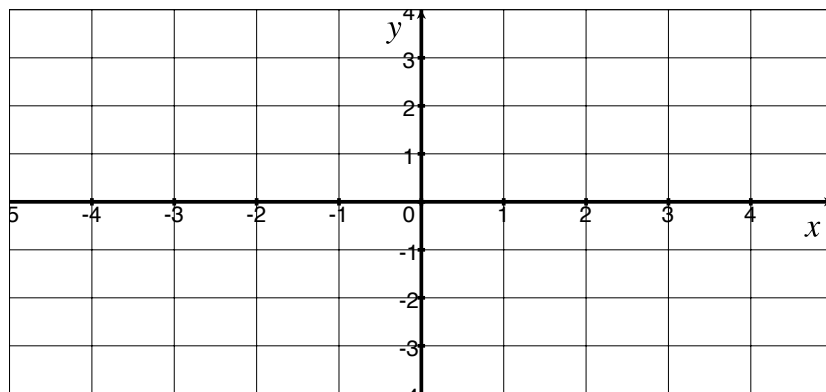(a) [10] Find the following (if they exist):

$g(-4)$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $g'(-4)$

$(g \circ f)'(-4)$ $\qquad\qquad\qquad\qquad\qquad$ $(\frac{f}{g})'(-4)$

(b) [3] *Sketch* the graph of f'.

4. If f is a function defined on the interval $[-10, 10]$, explain in elementary terms what exactly $f'(3)$ is.

5. Find $\dfrac{dy}{dx}$ for each of the following:
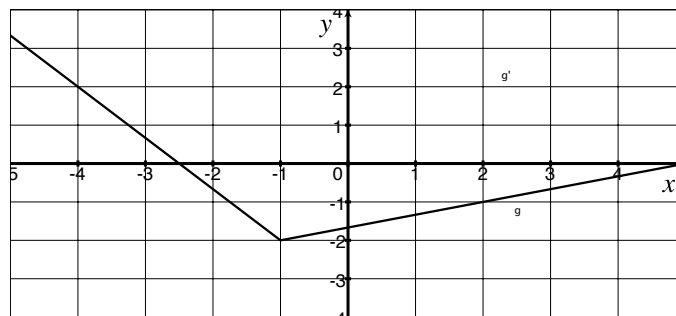
(§3.1 #22) $\quad y = \sqrt{\dfrac{1}{x^3}}$

(§3.2 #5) $\quad y = 2^x + \dfrac{2}{x^3}$

(§3.3 #17) y=$5e^{5x+1}$

(§3.4 #17) $y = x \ln(2x + 1)$

6. If f is a function defined on the interval $[-10, 10]$, explain in elementary terms what exactly $\int_{-10}^{3} f(x)\,dx$ is.

7. Let g be the function whose graph is given below.

(a) Approximate $\int_{-4}^{2} g(x)\,dx$ using left-hand approximations and six rectangles.

(b) Find $\int_{-4}^{2} g(x)\,dx$ exactly.

8. *Carefully* write down the Fundamental Theorem of Calculus.

9. Find the following:

   ($\S 7.3$ #18) $\displaystyle\int_0^{\frac{\pi}{4}} \sin t + \cos t \, dt$ ($\S 7.3$ #13) $\displaystyle\int_1^2 \frac{1}{x} \, dx$

10. Let $f(x) = -(x+1)(x-2)(x-4)$

    (a) Find $\displaystyle\int_{-1}^4 f(x) \, dx$

    (b) Find the *total* area (not signed) bounded between the graph of f and the x-axis.

11. The total cost to produce q hundred units is $C(q) = q^2 \ln(q) - q \sin(q) + 2$.

   (a) Find the cost of producing 150 units.

   (b) Find the average cost of producing 150 units.

   (c) Find the marginal cost of producing 150 units.

12. [8] A commercial cattle ranch currently allows 20 steers per acre of grazing land; on the average its steers weight 2000 lb at market. Estimates by the Agriculture Department indicate that the average market weight per steer will be reduced by 50 lbs for each additional steer added per acre of grazing land. How many steers per acre should be allowed in order for the ranch to get the largest possible total market wight for its cattle?

13. An offshore oil well is located in the ocean at a point W, which is 5 miles from the closest shorepoint A on a straight shoreline. The oil is to be piped to a shorepoint B that is 8 miles from A by piping it on a straight line under water from W to some shorepoint P between A and B ad then on to B via a pipe along the shoreline. If the cost of laying pipe is $100,000 per mile under water and $75,000 per mile over land, where should the point P be located to minimize the cost of laying the pipe?