EXAM 2 Key

TQS 211

Practice

Note: This is a practice midterm and is intended only for study purposes. The actual exam will contain different questions and perhaps a different layout.

1. [] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be functions, and x and y be real numbers.

   T F $(2f)'(x) = 2f'(x)$

   T F $\cos(x + y) = \cos(x) + \cos(y)$

   T F If $f'(a) < 0$, then the graph of $f(x)$ is decreasing when $x = a$.

   T F When $MC = MR$ the company may be maximizing profit.

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [] For each rule of f given below, find $f'(x)$.

$$f(x) = \sin(x) + \frac{3}{x}$$

$(\sin(x) + 3x^{-1})' = [\sin(x)]' + [3x^{-1}]'$

$= \cos(x) + 3[x^{-1}]'$

$= \cos x - 3x^{-2}$

$$f(x) = 2x^2 + \ln(7x^2)$$

$(2x^2 + \ln(7x^2))' = (2x^2)' + (\ln(7x^2))'$

$= 2(x^2)' + [\ln(7x^2)]'$

$= 2 \cdot 2x + \frac{2}{x}$

$f(x) = \ln x \quad g(x) = 7x^2$

$f'(x) = \frac{1}{x} \quad g'(x) = 14x$

$f'(g(x))g'(x) = f'(7x^2) \cdot 14x = \frac{14x}{7x^2}$

$$f(x) = \frac{3^x \cos(x)}{\sin(x)}$$

$[3^x \cos(x)]' = 3^x(\cos(x))' + (3^x)'\cos(x)$

$= 3^x(-\sin x) + 3^x \ln(3)\cos(x)$

$f'(x) = \frac{\sin(x)[3^x \cos x]' - [3^x \cos(x)](\sin(x))'}{(\sin(x))^2}$

$= \frac{\sin(x)[-3^x \sin x + 3^x \cos(x)\ln 3] - 3^x \cos x \cos x}{\sin^2(x)}$

$$f(x) = (\sqrt{3x^4 - x})(e^x - 4)$$

$f'(x) = (3x^4 - x)^{\frac{1}{2}}[e^x - 4]' + (e^x - 4)([3x^4 - x]^{\frac{1}{2}})'$

$= (3x^4 - x)^{\frac{1}{2}} e^x + (e^x - 4)[(3x^4 - x)^{\frac{1}{2}}]'$

$= e^x(3x^4 - x)^{\frac{1}{2}} + (e^x - 4)\frac{1}{2}[3x^4 - x]^{-\frac{1}{2}}[12x^3 - 1]$

$f(x) = x^{\frac{1}{2}} \quad g(x) = 3x^4 - x$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \quad g'(x) = 12x^3 - 1$

$f'(g(x))g'(x) = f'(3x^4 - x)[12x^3 - 1]$

$$f(x) = \frac{6x^4 - x^{\frac{1}{3}}}{\sqrt{x}} = \frac{6x^4 - x^{\frac{1}{3}}}{x^{\frac{1}{2}}}$$

$\frac{1}{3} - \frac{1}{2}$

$\frac{2}{6} - \frac{3}{6} = -\frac{1}{6}$

$= 6x^{3.5} - x^{\frac{1}{3} - \frac{1}{2}} = 6x^{3.5} - x^{-\frac{1}{6}}$

$f'(x) = 6 \cdot 3.5x^{2.5} + \frac{1}{6}x^{-\frac{7}{6}}$

$$f(x) = (-4x + 7)^{12}(-2x^2 - 3x)^9$$

$f'(x) = [(-4x + 7)^{12}]'(-2x^2 - 3x)^9 + (-4x + 7)^{12}[(-2x^2 - 3x)^9]'$

$f(x) = x^{12} \quad g(x) = -4x + 7$

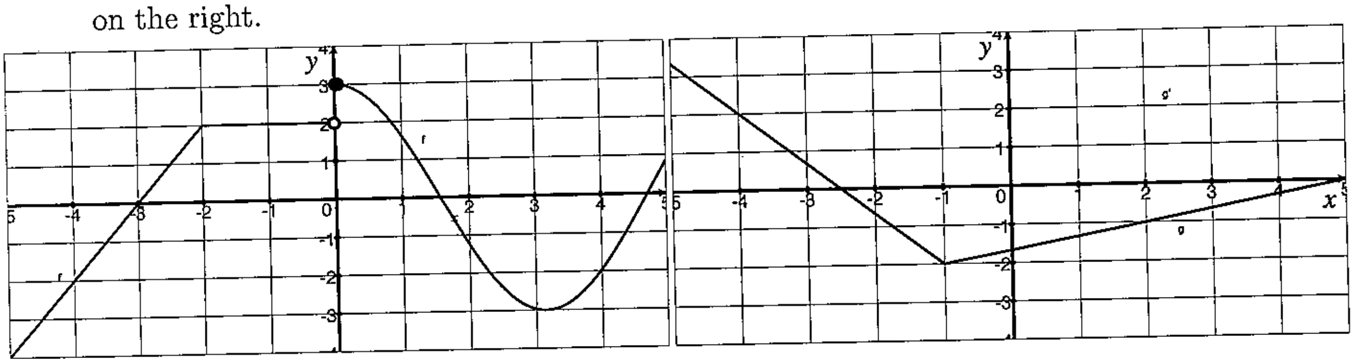
$f'(x) = 12x^{11} \quad g'(x) = -4$

$f(x) = x^9 \quad g(x) = -2x^2 - 3x$

$f'(x) = 9x^8 \quad g'(x) = -4x - 3$

$f'(x) = 12(-4x + 7)^{11}[-4](-2x^2 - 3x)^9 + (-4x + 7)^{12}\, 9(-2x^2 - 3x)^8[-4x - 3]$

3. Let f be the function whose graph is on the left and g be the function whose graph is on the right.



(a) [10] Estimate the following (if they exist):

$f'(-3) = 2$

$\left(\frac{f}{g}\right)'(-4) = \frac{g(-4)f'(-4)-f(-4)g'(-4)}{[g(-4)]^2}$

$= \frac{(\cancel{2}\tfrac{4}{3})(2)-(-2)(-\frac{4}{3})}{2^2} = \frac{4-\frac{8}{3}}{2}$

$(f \cdot g)'(-3) = f(-3)g'(-3)+f'(-3)g(-3)$

$= 0\cdot(-\frac{4}{3}) + 2\cdot\frac{2}{3}$

$= \frac{4}{3}$

$(f \circ g)'(2) = f'(g(2))g'(2)$

$= f'(-1)\,\frac{1}{3}$

$= 0 \cdot \frac{1}{3} = 0$

4. The oil spill in the gulf is being fed by a well that produces approximately ??? cubic meters per day. Assume for now that the oil spill is approximately circular and the thickness of the oil is uniformly half a meter thick.

(a) Write down a relationship between the total volume of the oil and the radius of the oil spill.

Vol of a cylinder = height · area of circle $= .5 \cdot \pi r^2 = V$

(b) What rate is the volume of the oil cylinder expanding per hour?

whatever the ??? above was
(should have been a # had I remembered to look it up)

(c) How fast is the radius of the oil spill changing when the oil spill is 200 meters across?

ie $\left.\frac{dr}{dt}\right|_{r=200} = ?$

$V = .5\pi r^2$

$\frac{dV}{dt} = \frac{d}{dt}(.5\pi r^2)$

$\frac{dV}{dt} = .5\pi \frac{d}{dt}(r^2)$

$\frac{dV}{dt} = .\cancel{5}\pi\, \cancel{2}r \frac{dr}{dt}$

$\frac{dV}{dt} = \cancel{.5}\pi r \frac{dr}{dt}$

so solve for $\frac{dr}{dt}$ when

$??? = \pi \cdot 200 \frac{dr}{dt}$

5. There is a "Rule of 70" or "Rule of 7" that commonly arrises in economic or financial circles. The rule is as follows: The time is takes your money to double at an interest rate of r is approximately $\frac{70}{r}$. We will find out where this rule comes from.

(a) Assume you have P_0 dollars to invest. You find an investment that promises an effective annual interest rate of $r\%$. Write down a function that describes how much money you have after t years. (This is a throw back from §1.5.)

$P_0(1+r)^t$ = \$ after t years.

(b) We want to know when the investment doubles, that is, find t so that you have a total of $\$2P_0$.

$$\frac{2P_0}{P_0} = \frac{P_0(1+r)^t}{P_0}$$

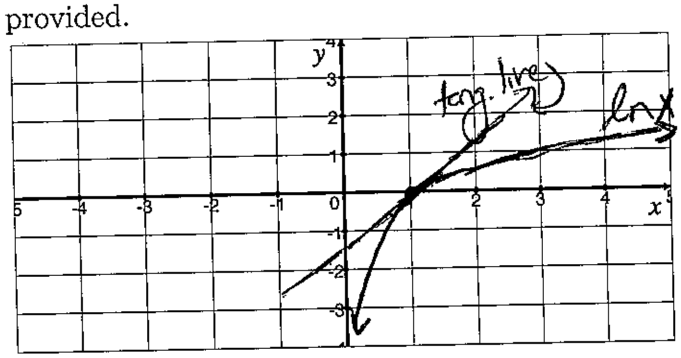
$$2 = (1+r)^t$$

$$\ln 2 = \ln(1+r)^t$$

$$\ln 2 = t\ln(1+r)$$

$$\frac{\ln 2}{\ln(1+r)} = t$$

(c) Draw the graph of $\ln x$ on the axis provided.



(d) Find the equation of the line that is tangent to the graph of $\ln x$ at $x = 1$.

looking for $y = mx + b$

m = slope of line tangent to ln x when x = 1 $= \frac{1}{x}\Big|_{x=1} = 1$

passes through $(1,0)$ so

$0 = 1 \cdot 1 + b \Rightarrow b = -1$

Thus $y = x - 1$

(e) Use the line you found in (d) to approximate the function $\ln x$ when x is near 1 to simplify your answer in (b). Note, since r is usually closer to .05 than to .95, we can think of $1 + r$ as a number close to 1.

$\ln(1+r) \approx (1+r) - 1 = r$

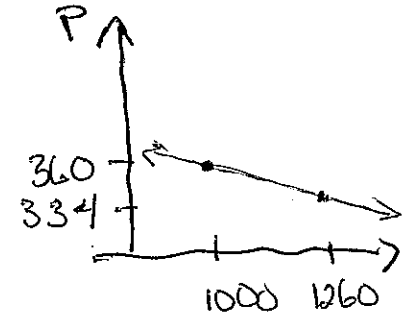
so

$$t = \frac{\ln 2}{\ln(1+r)} \approx \frac{\ln 2}{r} \approx \frac{.70}{r}$$

:)

6. [] A manufacture has been selling 1000 televisions a week at \$360 each. A market survey indicates that for each \$26 rebate offered to a buyer, the number of sets sold will increase by 260 per week. Let q be the number of televisions demanded and p be the price.

(a) Assume the relationship between the demand q and the price p is linear. Express p as a function of q. looking for $y = mx + b$



slope $= \frac{-26}{260} = \frac{-1}{10}$

passes through $(1000, 360)$ so

$360 = 1000 \cdot {-1/10} + b$

$360 = -100 + b$

$460 = b$

So $p = -1/10\, q + 460$

(b) Use the work from above to express the revenue, R, as *only* a function of q.

$R = p \cdot q$

$= (-1/10\, q + 460) q = -1/10\, q^2 + 460 q.$

(c) If the weekly cost function if $60000 + 120q$, where q is the number of television sets sold per week, how should it set the size of the rebate to maximize its profit?

Max profit may happen when MC = MR

$MR = (-1/10\, q^2 + 460 q)' = -1/10\, 2q + 460 = -1/5\, q + 460$

$MC = (60000 + 120 q)' = 120$

find q when MR = MC

$-1/5\, q + 460 = 120$

$-1/5\, q = -340$

$q = 5 \cdot 340 = 1700$ # of TV's sold to max profit.

$\begin{array}{r} 460 \\ 120 \end{array}$ $\quad \begin{array}{r} 34 \\ 5 \\ \hline 170 \end{array}$

Thus we want price to be $-1/10(1700) + 460 = -170 + 460 = \$290$

=> set the rebate to \$360 − 290 or \$70