

Note: This is a practice midterm and is intended only for study purposes. The actual exam will contain different questions and perhaps have a different layout.

1. [] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T $\frac{3x+y}{3z} = \frac{x+y}{z}$

T $(x+y)^2 = x^2 + y^2$

T $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ for all a even if the den. had a 0 in it,
this would be false b/c we need $\lim_{x \rightarrow a} g(x) \neq 0$

^{Cost} T No profit is made when $MR < MC$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

- a. here $MC > MR$ but profit is still had.
2. Find $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{(x-3)(x+1)}$ using properties of limits.

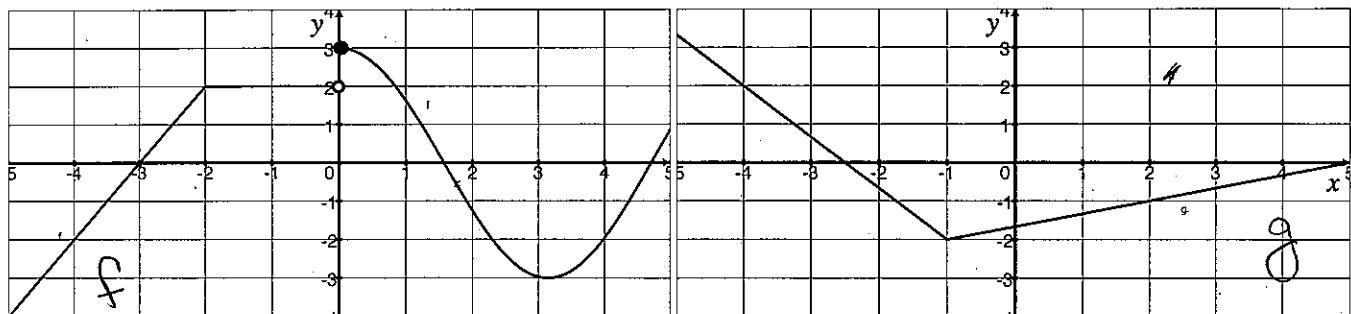
$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{(x-3)(x+1)} = \lim_{x \rightarrow 3} \frac{(x-3)(x-1)}{(x-3)(x+1)}$$

$$= \lim_{x \rightarrow 3} \frac{x-1}{x+1}$$

$$= \frac{\lim_{x \rightarrow 3} (x-1)}{\lim_{x \rightarrow 3} (x+1)} \quad \text{b/c } \lim_{x \rightarrow 3} (x+1) \neq 0$$

$$= \frac{\lim_{x \rightarrow 3} x - \lim_{x \rightarrow 3} 1}{\lim_{x \rightarrow 3} x + \lim_{x \rightarrow 3} 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

3. Let f be the function whose graph is on the left and g be the function whose graph is on the right.



- (a) [10] Find the following (if they exist):

$$\lim_{x \rightarrow -2} f(x)$$

2

$$\lim_{x \rightarrow -1} \left(\frac{g(x)}{5} - 2 \right) = \lim_{x \rightarrow -1} \frac{g(x)}{5} - \lim_{x \rightarrow -1} 2$$

$$= \frac{-2}{5} - 2 = \boxed{\frac{-12}{5}}$$

$$\lim_{x \rightarrow 0} 2f(x)$$

does not exist

$$g(-4)$$

2

$$(g \circ f)(-4)$$

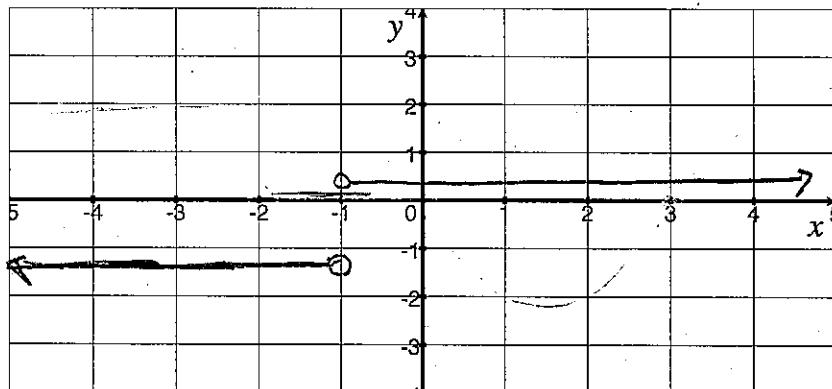
$$g'(-4) \approx -\frac{4}{3}$$

$$g(-2) \approx -\frac{1}{2}$$

- (b) Find all the x values that f is discontinuous.

when $x = 0$

- (c) [3] Sketch the graph of g' .



$-\frac{4}{3}$

$\frac{1}{3}$

4. The demand curve for a product is given by $q = 300 - 3p$, where p is the price of the product and q is the quantity consumers will buy at that price.

- (a) [2] Write the revenue as a function of *only* price (there should be no q 's).

$$Rev = p \cdot q = p(300 - 3p) = 300p - 3p^2$$

- (b) [3] Find the marginal revenue when the price is \$10, and interpret your answer in terms of revenue.

$$\begin{aligned} MR &= R' = \lim_{h \rightarrow 0} \frac{R(p+h) - R(p)}{h} = \lim_{h \rightarrow 0} \frac{300(p+h) - 3(p+h)^2 - [300p - 3p^2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{300p + 300h - 3p^2 - 6ph - 3h^2 - 3p^2 + 3h^2}{h} = \lim_{h \rightarrow 0} 300 - 6p - 3h \end{aligned}$$

- When the price is \$10
the approx revenue of producing 1 more unit will increase Revenue by \$240.
- (c) [4] If the marginal cost of making the product is \$20, and the business has the ability to set the price (by controlling q), what should the business set the price to so as to maximize profit?

Profit is maximal when $MR = MC$

From above $MR(p) = 300 - 6p$.

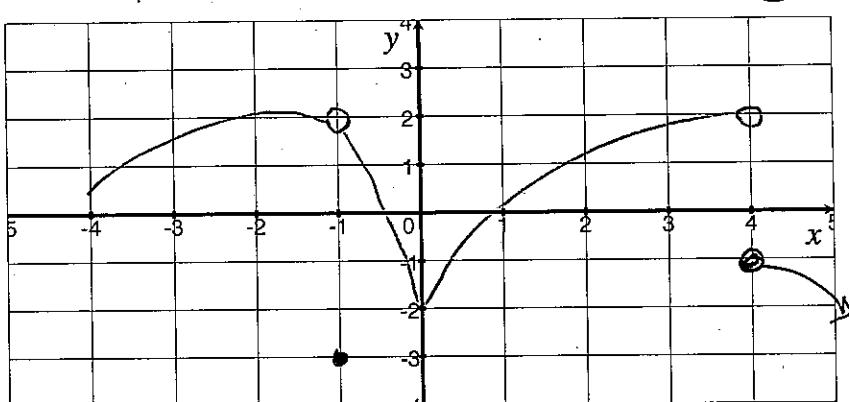
$$So \quad MC = MR \Rightarrow 20 = 300 - 6p$$

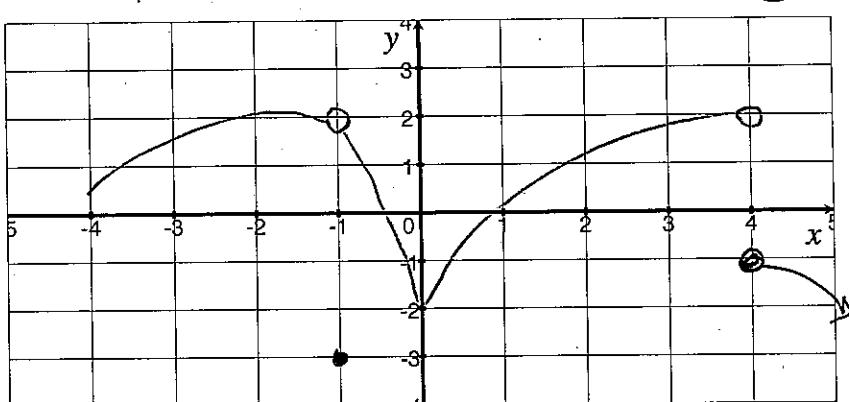
$$\Rightarrow 6p = 280$$

$$\Rightarrow p = \frac{280}{6} = \frac{140}{3} \approx \$46.67$$

5. Sketch a graph of a function α that satisfies all of the following:

$$\alpha(-1) = -3, \lim_{x \rightarrow -1} \alpha(x) = 2, \alpha \text{ is not continuous at } x = 4, \text{ and for all } x > 0, \alpha''(x) < 0.$$

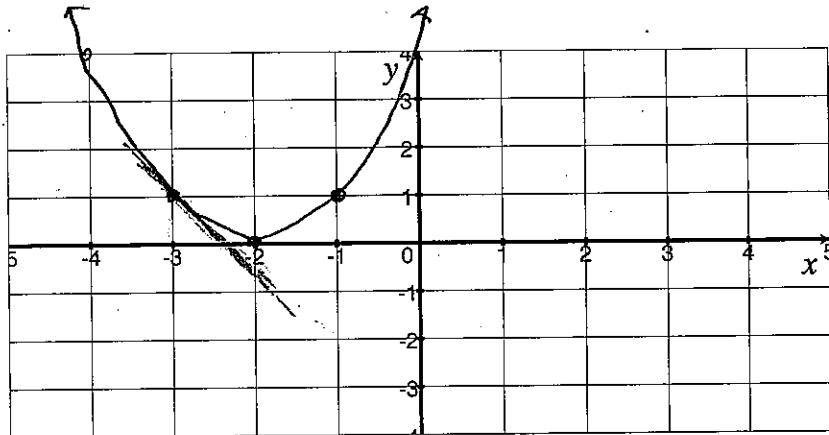
jump.



6. Let $m(x) = (x + 2)^2$.

(a) [1] Carefully graph f .

Shift left 2 units.



(b) Estimate $m'(-3)$.

$$\text{From the graph } \approx -\frac{1.5}{1} = -1.5$$

(c) Find $m'(-3)$ algebraically.

$$\begin{aligned} m'(-3) &= \lim_{h \rightarrow 0} \frac{m(-3+h) - m(-3)}{h} = \lim_{h \rightarrow 0} \frac{(-3+h+2)^2 - (-3+2)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{(h-1)^2 - (-1)^2}{h} = \lim_{h \rightarrow 0} \frac{h^2 - 2h + 1 - 1}{h} \\ &= \lim_{h \rightarrow 0} h - 2 = \lim_{h \rightarrow 0} h - \lim_{h \rightarrow 0} 2 \\ &= 0 - 2 = -2 \end{aligned}$$

(d) Draw the line tangent to the graph of m at $x = -3$.

(e) Find an equation for the line tangent to the graph of m at $x = -3$.

we want to find $y = mx + b$
 \uparrow
 slope \uparrow
 $y = mx + b$

$m'(-3)$ is the slope
 of the line I've
 drawn so I have
 that.

$$\rightarrow y = -2x + b$$

to find b not the line
 passes through the point
 $(-3, 1)$ so

$$1 = -2(-3) + b \Rightarrow b = -5$$

Thus our equation is $-2x - 5 = y$

7. A company's cost of producing q liters of a chemical is $C(q)$ dollars; this quantity can be sold for $R(q)$ dollars. Suppose $C(2000) = 5930$ and $R(2000) = 7780$.

- (a) What is the profit at a production level of 2000?

$$\text{Profit} = \text{Rev} - \text{Cost}$$

$$\text{So Profit at 2000 units is } 7780 - 5930 = 1750$$

- (b) When production is increased to 2001 the total cost is \$5930.10 and total revenue is \$7782.5. Estimate $MC(2000)$ and $MR(2000)$.

(Type)

$$MC(2000) \approx R(2001) - C(2000)$$

$$= 5930.10 - 5930$$

$$= .10$$

$$MR(2000) \approx R(2001) - R(2000)$$

$$= 7782.5 - 7780$$

$$= 2.5$$

- (c) If $MC(2000) = 2$ and $MR(2000) = 2.5$, what is the approximate change in profit if q is increased from 2000 to 2010?

$$\text{Rev}(2010) \approx \text{Rev}(2000) + \Delta \text{Rev}$$

$$\text{Cost}(2010) \approx \text{Cost}(2000) + \Delta \text{Cost}$$

$$= 7780 + 2.5 \cdot 10$$

$$= 5930 + 2 \cdot 10$$

$$= 7780 + 25$$

$$= 5930 + 20$$

$$= 7805$$

$$= 5950$$

- (d) Should the company increase or decrease production from $q = 2000$?

$$\approx 1855$$

increase b/c $MR > MC$

\rightarrow Change in
Profit is

$$\$105$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{2}{x+h-3} - \frac{2}{x-3}}{h} = \lim_{h \rightarrow 0} \frac{\frac{2(x-3) - 2(x+h-3)}{(x-3)(x+h-3)}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{2x-6-2x-2h+6}{(x-3)(x+h-3)} \div \frac{1}{h} \right] = \lim_{h \rightarrow 0} \left[\frac{-2h}{(x-3)(x+h-3)} \cdot \frac{1}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{(x-3)(x+h-3)} = \lim_{h \rightarrow 0} \frac{-2}{(x-3)(x+h-3)}$$

$$= \frac{\lim_{h \rightarrow 0} -2}{\lim_{h \rightarrow 0} (x-3)(x+h-3)} = \frac{-2}{\lim_{h \rightarrow 0} (x-3) \lim_{h \rightarrow 0} (x+h-3)} = \frac{-2}{(x-3)(x+0-3)}$$

$$= \frac{-2}{(x-3)(x-3)} = \frac{-2}{(x-3)^2}$$