Integrals below the x-axis

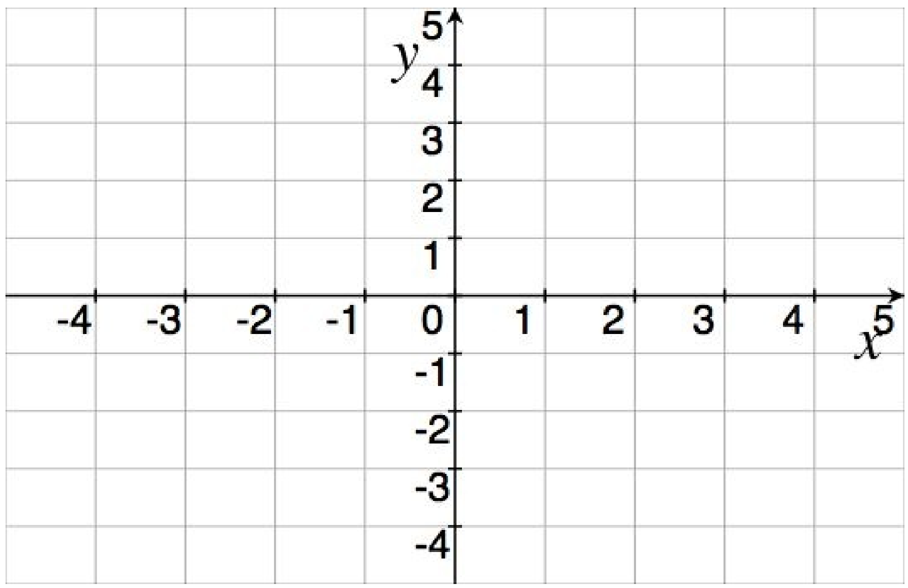
Recall $\int_a^b f(x)\,dx$ corresponds to the area 'below' the graph of f from a to b marked on the horizontal axis. More precisely, $\int_a^b f(x)\,dx$, is the area bounded by the graph of f, the horizontal axis, and the two vertical lines $x = a$ and $x = b$.

To approximate $\int_a^b f(x)\,dx$, we use rectangles with width Δx and a height determined by f. For example if we used the left hand approximation with two rectangles we would compute:

$$\int_2^4 f(x)\,dx = f(2)\Delta x + f(3)\Delta x$$

1. Let $f(x) = -\frac{1}{2}x + 1$.

   (a) Draw f.

   (b) What is $f(3)$?

   (c) Use left hand approximation with two rectangles to compute $\int_2^4 f(x)\,dx$.

   Point: If f is below the horizontal axis from $[a, b]$, $\int_a^b f(x)\,dx$ returns the area bound between the horizontal axis, the graph of f, and the vertical lines $x = a$ and $x = b$, but with a *negative sign.*

   (d) Find $\int_2^4 f(x)\,dx$ exactly.

   (e) Find $\int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx$ exactly. What is the relationship between your calculations in part (d) and part (e)?

If we think of $\int_2^4 f(x)\,dx$ as an area, we can convince ourselves rather quickly that

$$\int_2^4 f(x)\,dx = \int_2^b f(x)\,dx + \int_b^4 f(x)\,dx$$

where b is a number between 2 and 4. Much more can be shown if we look closer but lets leave it there for now.

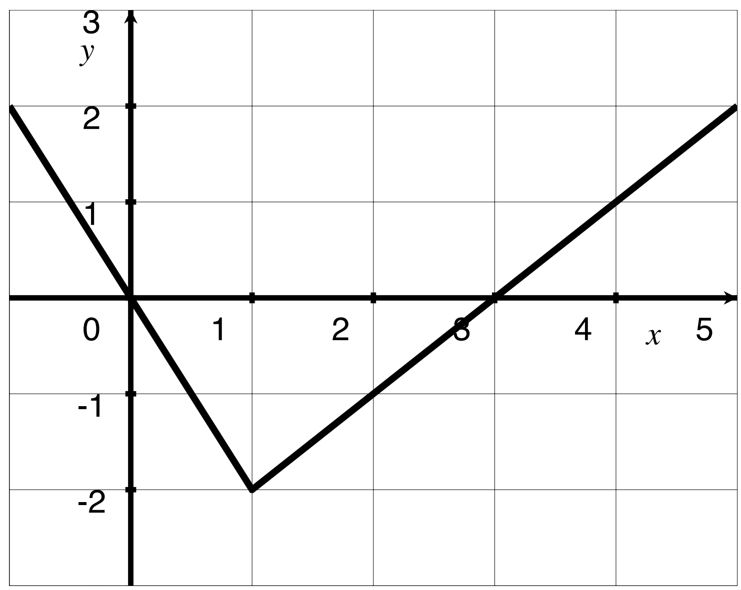
2. Let g be the function whose graph is below. You may want to use the above observations to find (exactly):

   (a) $\int_0^2 g(x)\,dx$

   (b) $\int_0^3 g(x)\,dx$

   (c) $\int_3^4 g(x)\,dx$

   (d) $\int_0^4 g(x)\,dx$



3. Given the graph of h, determine if $\int_0^5 h(x)\,dx$ is positive, negative, or about zero.