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10:12
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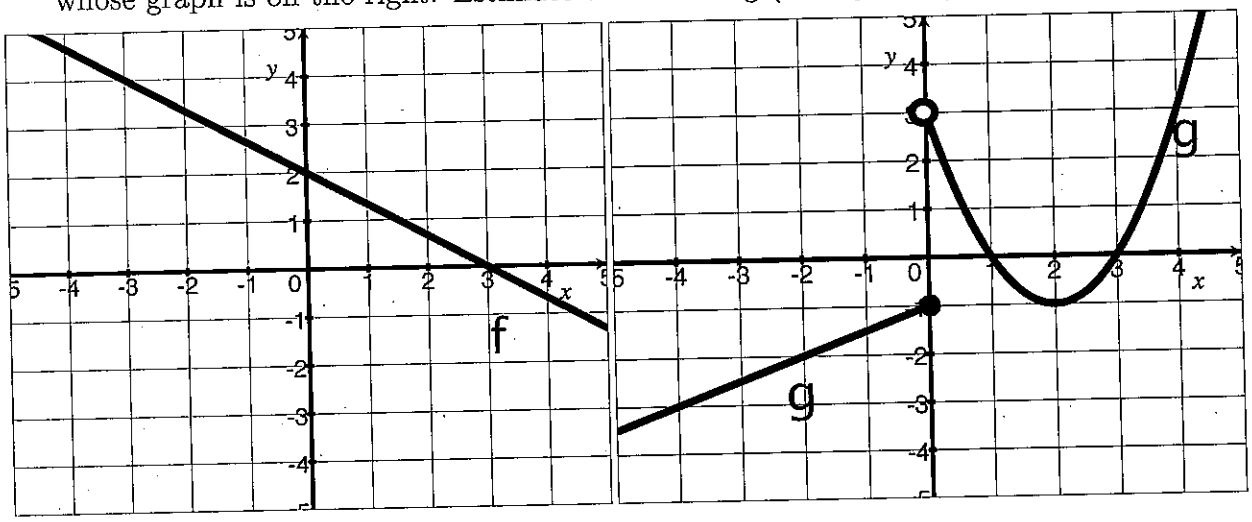
EXAM 2 Key

TQS 211

Spring 2010

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

1. (§3.4 #43) Let f be the function whose graph is on the left and g be the function whose graph is on the right. Estimate the following (if they exist):



(a) [3] $\left(\frac{f}{g}\right)'(2)$ $= \dfrac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \dfrac{(-1)(-2/3) - \cancel{\tfrac{1}{2}\cdot 0}}{(-1)^2}$ } .5 } +.5

$= \dfrac{2/3}{1} = 2/3$

quotient rule +.1
used correctly +.5
alg .5

(b) [3] $(f \cdot g)'(2)$ $= f'(2)g(2) + f(2)g'(2) = \underbrace{(-2/3)(-1)}_{+.5} + \underbrace{\tfrac{1}{2}\cdot 0}_{.5}$

$= 2/3$

product rule +.1
used correctly +.5
alg +.5

(c) [3] $(f \circ g)'(-3)$

$= f'(g(-3))\cdot g'(-3) = \underbrace{f'(-2.5)}_{+.5}\underbrace{(2/4)}_{+.5}$

$= (-2/3)(1/2) = -1/3$

chain rule +.1
used correctly +.5
alg +.5

2. [4 each] For each rule of f given below, find $f'(x)$. Note, you do *not* need to simplify.

(a) (§3.1 #27) $f(x) = \sqrt{x}(x+1) = x^{\frac{1}{2}}(x+1) = x^{3/2} + x^{\frac{1}{2}}$

$\underbrace{\tfrac{3}{2}x^{\frac{1}{2}}}_{+1} + \underbrace{\tfrac{1}{2}x^{-1/2}}_{+1}$

or

$(\sqrt{x})(x+1)' + (\sqrt{x})'(x+1)$

$\underbrace{x^{\frac{1}{2}} \cdot 1}_{+1} + \underbrace{\tfrac{1}{2}x^{-1/2}(x+1)}_{+1}$

~~power~~ product rule +1

alg/notation +1

simplified alg before +1

sum/notation +1

(b) (§3.2 Example 2) $f(x) = 5\ln x + 7e^x - 4x^2 + 12$

$\underbrace{5 \cdot \tfrac{1}{x}}_{+1} + \underbrace{7 \cdot e^x}_{+1} - 8x$

sum/diff +1

power rule/const +1

(c) (WebHW6 #10) $f(x) = \sin(\sin(x))$

$f(x) = \sin x \quad g(x) = \sin x$

$f'(x) = \cos x \quad g'(x) = \cos x$ } +2

$f'(g(x))g'(x) = f'(\sin x) \cdot \cos x$

$= \cos(\sin x)\cos x$

chain +1

used correctly +.5

notation +.5

(d) (§3.4 #27) $f(x) = \dfrac{3x + x^2}{5 + x}$

$$\frac{(5+x)(3x+x^2)' - (3x+x^2)(5+x)'}{(5+x)^2}$$

$$\frac{(5+x)\overbrace{(3+2x)}^{+1} - (3x+x^2)\overbrace{(1)}^{+1}}{(5+x)^2} \rightsquigarrow \frac{x^2+10x+15}{x^2+10x+25}$$

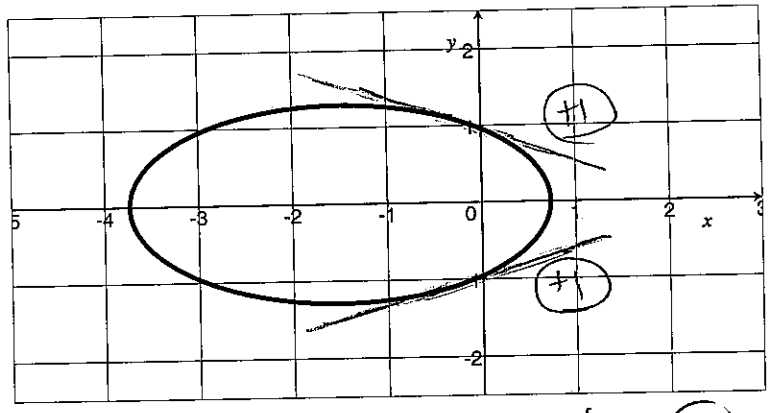
quotient +1

used correctly +.5

notation +.5

3. The graph of $3y^2 + (x + \frac{3}{2})^2 = 5$ is given below.

(a) [2] Sketch any lines that are tangent to the graph when $x = -\frac{1}{2}$.



(b) (Implicit Diff. Wks) [5] Find the slopes of the lines your drew for part (a).

want $\left.\frac{dy}{dx}\right|_{x=-1/2}$

implicit/diff both sides (+.5)

more corr

$\underbrace{6y}_{+.5}\underbrace{\frac{dy}{dx}}_{+.5} + \underbrace{2(x+3/2)}_{+.5} = \underbrace{0}_{+.5}$

$\frac{dy}{dx} = \frac{-2(x+3/2)}{6y}$

happens when $x = -1/2$ dy +.5 dx +.5 +.5 bothsi alg

is so that

$3y^2 + (-\frac{1}{2}+\frac{3}{2})^2 = 5$

$3y^2 + 1^2 = 5$ (+1)

$3y^2 = 4 \Rightarrow y = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$

the slopes are

$\frac{-2(-\frac{1}{2}+\frac{3}{2})}{6(\frac{2}{\sqrt{3}})}$ and $\frac{-2(-\frac{1}{2}+\frac{3}{2})}{6(\frac{-2}{\sqrt{3}})}$

(c) [3] Find the x coordinates a, so that the line tangent to the above oval at $x = a$ is a horizontal line.

ie when does $\frac{dy}{dx} = 0$

understand (+1)

$0 = \frac{-2(x+3/2)}{6y}$ notation (+1)

$\Rightarrow x = -3/2$

alg (+1)

or

$\frac{-2}{6\frac{1}{\sqrt{3}}}$ plug in values (+1) $\frac{1}{-6\frac{1}{\sqrt{3}}}$

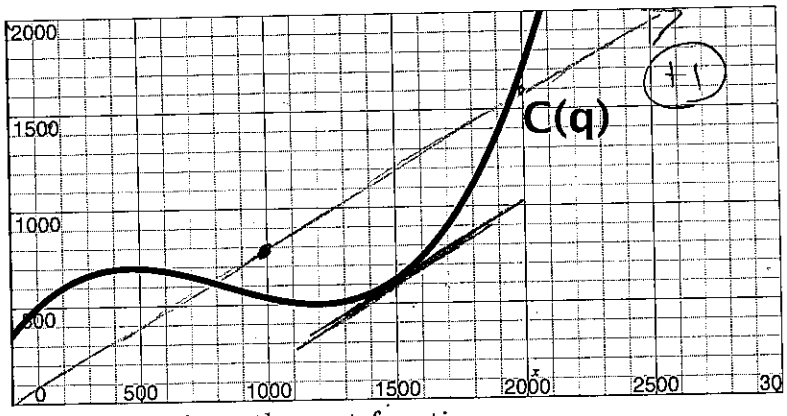
$\frac{-\frac{1}{2}\sqrt{3}}{6}$ $\frac{\sqrt{3}}{6}$

if guessed then +1

4. (5/6 Lecture) The cost of making q widgets is given by the function

$$C(q) = \left(\frac{q}{100} - 1\right)\left(\frac{q}{100} - 12\right)^2 + 500.$$

The graph of the cost function is provided for you below. The marked price for a widget is \$.80.



(a) [2] Find the revenue function if q widgets are sold.

$.8q$

(b) [1] *Carefully* draw the revenue function on the same axis as the cost function.

(c) [2] Use the graph to estimate when the company is breaking even.

$\approx$ 780 (+1) and 1950 (+1) items.

(d) (§3.1 #49) [3] Find the marginal revenue of producing the 1500th widget (be sure to tell me how you did it) and interpret your answer in terms of ~~costs~~ widgets & dollars

graphically $\approx$ 1 \$/item (+1) (+1)

If you increase production beyond 1500th widgets the additional widget will bring in about \$1

(e) 1 [2] Use the graph to estimate how many widgets should be sold to maximize profit.

1500 looks pretty good

(f) (a) Use calculus to find how many widgets should be sold to maximize profit?

When $MR = MC$ (+1)

$MR = .8$ (+1)

$$MC = \left(\frac{q}{100} - 1\right)\underbrace{2\left(\frac{q}{100} - 12\right)\frac{1}{100}}_{(+1)} + \underbrace{\frac{1}{100}\left(\frac{q}{100} - 12\right)^2}_{(+.5)}$$

product (+.5)
notation (+.5)
used chain (+.5)

So $MR = MC$

$$.8 = \left(\frac{q}{100} - 1\right)\frac{1}{50}\left(\frac{q}{100} - 12\right) + \frac{1}{100}\left(\frac{q}{100} - 12\right)^2$$

solve for q ... Alg (+1)