

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

1. (Limits Worksheet) Let  $f(x) = \frac{-3x^2 - 6x}{x + 2}$ .

- (a) [3] Estimate  $\lim_{x \rightarrow -2} f(x)$  either numerically or graphically. State which method you used and provide either calculations or a graph to support your answer.

$$\approx \frac{-3(-2.1)^2 - 6(-2.1)}{-2.1 + 2}$$

(2) # close to  
(1) plug into f  
(3) stated

- (b) [3] Use algebra and properties of limits to find  $\lim_{x \rightarrow -2} f(x)$  exactly.

$$\begin{aligned} \lim_{x \rightarrow -2} f(x) &= \lim_{x \rightarrow -2} \frac{-3x^2 - 6x}{x + 2} = \lim_{x \rightarrow -2} \frac{-3x(x+2)}{x+2} = \lim_{x \rightarrow -2} -3x \\ &= -3 \lim_{x \rightarrow -2} x = -3(-2) = +6. \end{aligned}$$

alg (1)  
correct limit (1)  
fix if limit (1.5) not short (1.5)

2. [6] (Practice Exam) Sketch a possible graph of a function  $\alpha$  that satisfies all of the following:

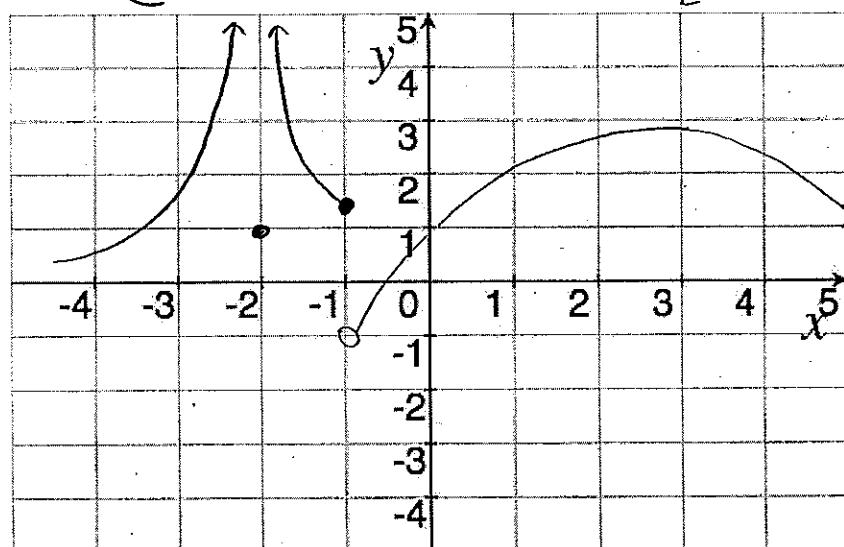
(a)  $\lim_{x \rightarrow -2} \alpha(x) = \infty$

(b)  $\alpha(-2) = 1$

(c)  $\alpha$  is not continuous at  $x = -1$ .

(d)  $\alpha'(x) = 0$  when  $x = 3$

(e)  $\alpha''(x) < 0$  when  $0 < x < 3$

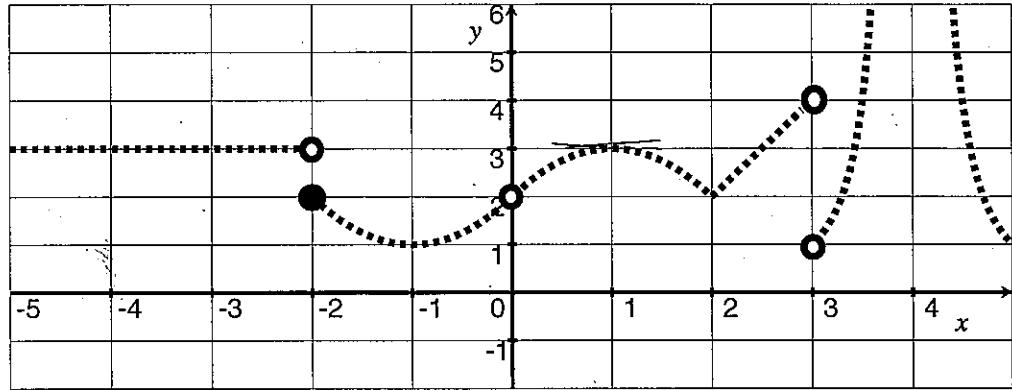


Symmetry (1)

not cont (1)

Concave down (1)

3. (Quiz 2) Let  $g$  be the piece-wise defined function below. This means the graph of  $g$  is the *entire* dotted graph shown below.



(a) [7] Estimate each of the following *if* it exists:

$$g(-4)$$

3

$$\lim_{x \rightarrow -3} g(x)$$

3

$$\lim_{x \rightarrow -2} g(x)$$

does not exist

$$\lim_{x \rightarrow 1} (5g(x) - 3)$$

$$g'(1)$$

$$g'(2.5)$$

$$5(3) - 3$$

$$15 - 3 = 12$$

alg prop +, S

0

see line traced +, S

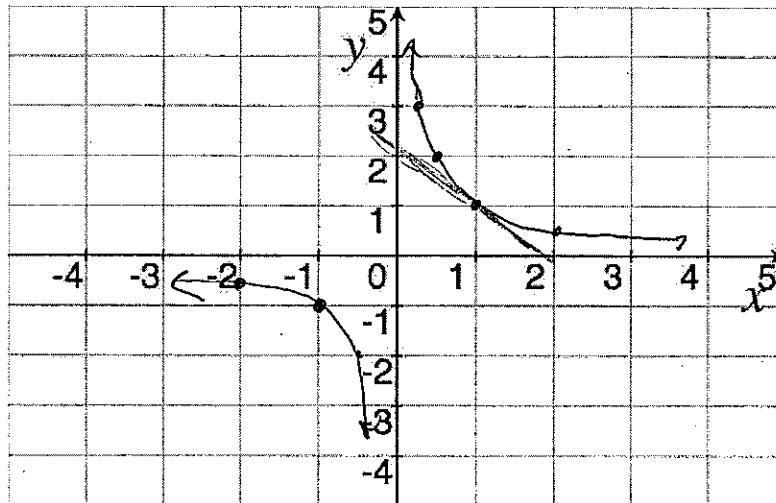
2

(b) [4] Find all  $x$  values that  $g$  is discontinuous.

@  $x = -2, 0, 3, 4$

4. Consider  $\beta(x) = \frac{1}{x}$ .

(a) [1] Carefully graph  $\beta$ .



(b) [1] Find the average rate of change of  $\beta$  from  $x = 1$  to  $x = 2$ .

$$\frac{\frac{1}{2} - 1}{2 - 1} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$$

(c) [1] Estimate  $\beta'(1)$ .

$$-1$$

(d) [4] Find  $\beta'(1)$  algebraically.

$$\lim_{h \rightarrow 0} \frac{\beta(1+h) - \beta(1)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{1+h} - \frac{1}{1}}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{1+h} - 1}{h}$$

+1.5      +1  
+1      +1

alg  
algebraic  
algebraic  
algebraic

$$= \lim_{h \rightarrow 0} \frac{-h}{1+h} \div h = \lim_{h \rightarrow 0} \frac{-h}{h(1+h)} = \lim_{h \rightarrow 0} \frac{-1}{1+h} = -1$$

took limit  
+1.5

(e) [1] Draw the line tangent to the graph of  $\beta$  at  $x = 1$ .

(f) [5] Find an equation for the line tangent to the graph of  $\beta$  at  $x = 1$  (ie the line that you drew in part (e)).

looking for  $y = mx + b$  (+1)  
 have  $m = -1$  of 4  
 $m = -1$   
 recognized (+1)

the point  $(1, 1)$  is on the line (+1)

$$\Rightarrow 1 = -1(1) + b \Rightarrow b = 1 + 1 = 2$$

plug in to find b (+1)

So  $y = -x + 2$

alg A.S  
got it  
+S

5. (§2.5 Worksheet) An industrial production process costs  $C(q)$  million dollars to produce  $q$  million units; these units then sell for  $R(q)$  million dollars. Assume  $C(2.1) = 5.1$ ,  $R(2.1) = 6.9$ ,  $MC(2.1) = 0.6$ , and  $MR(2.1) = 0.7$ .

- (a) [2] Explain what  $MR(2.1) = 0.7$  means in terms of production and dollars.

The revenue generated by increasing production from 2.1 to 3.1 million units is about 7 million dollars.

- (b) [1] Find the profit earned by producing 2.1 million units.

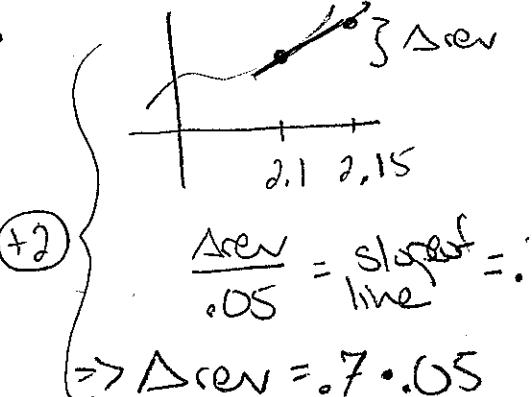
$$\text{Rev} - \text{Cost} = 6.9 - 5.1 = 1.8$$

- (c) [2] Should the company increase or decrease production? Why?

Increase b/c the additional items sold bring in more \$ than they cost.  
 right sense

- (d) [4] Estimate the total revenue when production is increased from 2.1 to 2.15 million units.

$$\begin{aligned}\text{Rev}(2.15) &\approx \overbrace{\text{Rev}(2.1)}^{+1} + \Delta\text{rev} \\ &= 6.9 + .7 \cdot .05 \\ &= 6.9 + .035 \\ &= 6.935\end{aligned}$$



- (e) [5] Estimate the total profit when production is increased from 2.1 to 2.15 million units.

Since Profit is Rev - Cost we have  $P(2.15)$  let us find it

$$\begin{aligned}C(2.15) &\approx C(2.1) + \Delta\text{cost} \\ &= 5.1 + .05 \cdot .6 \\ &= 5.1 + .03 \\ &= 5.13\end{aligned}$$

$$\begin{aligned}\text{Profit}(2.15) &\approx \\ &6.935 - 5.13 \\ &= 1.805 \\ &\text{million \$'s}\end{aligned}$$