Chain Rule Practice

Recall the chain rule: If f and g are both differentiable then $f \circ g$ is differentiable and

$$\frac{d}{dx}(f \circ g)(x) = f'(g(x))g'(x)$$

For each of the problems below complete the following:

1. Define function f and g so that:
   - both functions are easily differentiable *and*
   - the given function has the rule $f \circ g$.
2. Find f' and g'.
3. Find $\frac{d}{dx}(f \circ g)(x)$

Function	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$\frac{d}{dx}(f \circ g)(x)$
$\sqrt{x^3 - 5}$	$\sqrt{x}$	x^3-5	$\frac{1}{2}x^{-1/2}$	$3x^2$	$f'(g(x))g'(x)$ $= f'(x^3-5) \cdot 3x^2$ $= \frac{1}{2}(x^3-5)^{-1/2} \cdot 3x^2$
$(x^3 - 1)^{100}$	x^{100}	x^3-1	$100x^{99}$	$3x^2$	$f'(g(x))g'(x)$ $= f'(x^3-1) \cdot 3x^2$ $= 100(x^3-1)^{99} \cdot 3x^2$
$e^{3x^2 - x}$	e^x	$3x^2-x$	e^x	$6x-1$	$f'(g(x))g'(x)$ $= f'(3x^2-x)[6x-1]$ $= e^{3x^2-x}(6x-1)$
$\ln(5x - x^2)$	$\ln x$	$5x-x^2$	$\frac{1}{x}$	$5-2x$	$f'(g(x))g'(x)$ $= f'(5x-x^2) \cdot (5-2x)$ $= \frac{1}{5x-x^2}(5-2x)$

4. Notice that we can use the chain rule in conjunction with the previous rules we already learned. Use the work you did on the previous page to find the derivative of the following functions:

(a) $\left(\pi e^{3x^2-x}+e\right)' = \pi\left(e^{3x^2-x}\right)+(e)'$

$= \pi\left[\underbrace{e^{3x^2-x}}_{\text{from previous page.}}((6x-1))\right]+0$

(b) $\left[2^x+\ln(5x-x^2)\right]' = (2^x)'+\left[\ln(5x-x^2)\right]'$

$= 2^x\ln 2 + \underbrace{\frac{1}{5x-x^2}}_{\text{from previous page}}(5-2x)$

5. The chain rule can also be used in conjunction with itself. That is, we can use the chain rule to work on a derivative, but when trying to find the $g'(x)$, we may need to use the chain rule *again*.

Function	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$\frac{d}{dx}(f\circ g)(x)$
$\sqrt{47+3e^{3x^2-x}}$	$x^{\frac{1}{2}}$	$47+3e^{3x^2-x}$	$\frac{1}{2}x^{-\frac{1}{2}}$	$(47)'+3(e^{3x^2-x})'$ $0+3e^{3x^2-x}((6x-1))$	

$f'(g(x))g'(x)$

$= f'(47+3e^{3x^2-x})\cdot g'(x)$

$= \frac{1}{2}(47+3e^{3x^2-x})^{-\frac{1}{2}}\cdot 3e^{3x^2-x}(6$

$= 9((6x-1)e^{3x^2-x}(47+3e^{3x^2-x}$

yes you need to use the chain rule within itself.

$\ln(x+3^{x^3+7x})$

$f(x)=\ln x$

$g(x)=x+3^{x^3+7x}$

$f'(x)=\frac{1}{x}$

$g'(x)=(x)'+(3^{x^3+7x})'$

$=1+(3^{x^3+7x})' = 1+f'(g(x))g'(x) = 1+f'(x^3+7x)(3x^2+7)$

$f(x)=3^x \quad g(x)=x^3+7x$

$f'(x)=3^x\ln 3 \quad g'(x)=3x^2+7$

$= 1+3^{x^3+7x}\ln 3(3x^2+7$

So before: $\frac{1}{x+3^{x^3+7x}}\left(1+3^{x^3+7x}\ln 3(3x^2+7\right.$