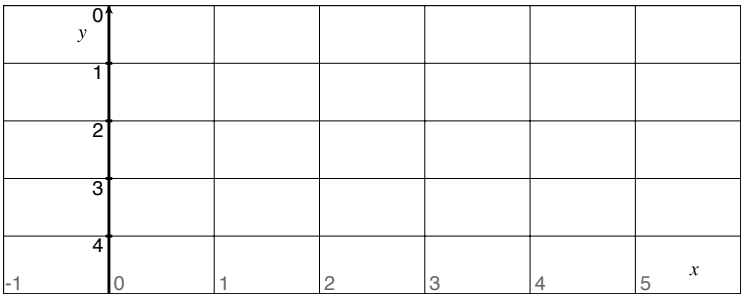
2^{nd} Derivatives

Get into groups of two or three people and work on the following problems.

1. The height (in feet) of a yoyo is well approximated for the first four seconds by the function:
   $p(t) = \frac{3}{8}x(x-2)(x-4)+3$.

   (a) Draw the height function.



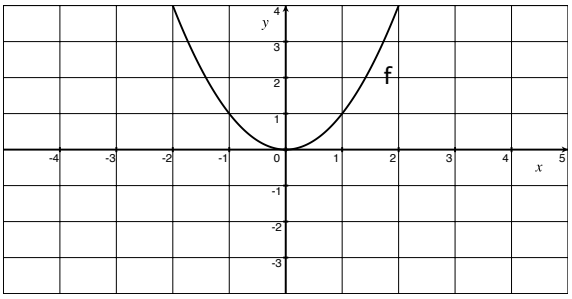
   (b) Find $p(4)$. What do the numbers mean in physical terms?

   (c) Estimate $p'(4)$. What do the numbers mean in physical terms (in particular, what does the sign mean)?

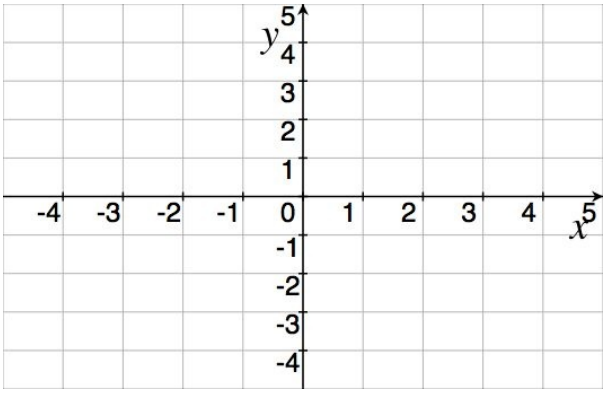
   (d) Use the above work to estimate $p(4.5)$.

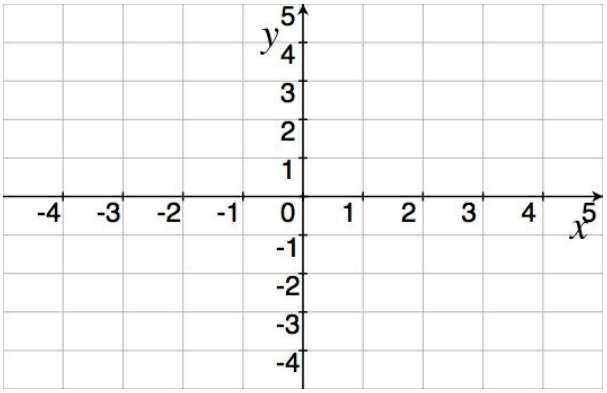
   (e) Find $p'(x)$ algebraically.

2. Let $f(x) = x^2$. Below is the graph of f.



(a) Find $f'(x)$ algebraically and then graph this function on the axis provided on the left below.





(b) Since f' is again a function we can take the derivative of f'. We call this new function f'' and say "f double prime". Find $f''(x)$ algebraically and then graph f'' on the axis above and to the right. Note whether $f''(x)$ is above or below the x-axis.

(c) Given a function g, what do you think $g'''(x)$ is? Find $f'''(x)$ for this problem.

3. Concavity is defined on page 17. The words are worth reading but the pictures are better. Draw 3 curves that are concave up and verify your answers with the definition on page 17.

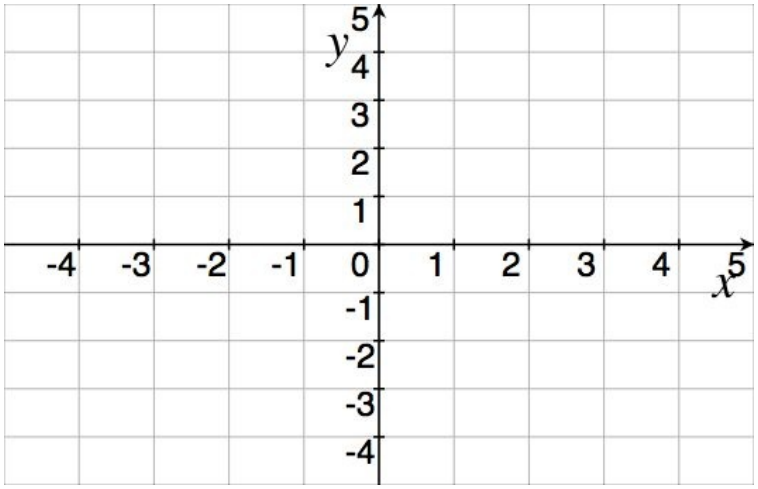
4. Is the function f in 2. concave up or concave down?

   There is a connection between concavity and the sign of the second derivative. Problem 2. is the best way to remember it:

   $f'' > 0$ on an interval means that graph of f is concave up.

   Verify the above claim is consistent with your work in 4.

5. Graph a function g that is concave down when $x < 0$ and concave up when $x > 0$. This means that the function g will have a *point of inflection* (the x value that a graph switches from concave up to concave down or vice versa). Note, you will need to take *two* derivatives of the function whose graph your draw so don't make it too crazy.



   (a) Sketch the curve of g'.

   (b) Sketch the curve of g''. What is the sign of f'' on the interval that g is concave down?

   Verify your conclusion by reading the box on page 119.