#5 most similar ≠ most common
= respect: 5+3=8+2=10

Key

Midterm **TMath 171** Winter 2025

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) and one 1-sided 8.5 in by 11 in sheet of notes.

1. [6] Let a, b, and c be whole numbers. Are the following statement always true, sometimes true, or never true? Briefly justify your answer.

   (a) (ExtraPractice§3.2 #17)

$$a - (b - c) = (a - b) - c$$

Start (+.5)
(+1) respect order ops

Sometimes (+.5)

(+.5) true eg: if $0=b=c$ then $a-(0-0)=(a-0)-0=a$

(+.5) not true eg: If $a=1$, $b=2$ and $c=3$
$a-(b-c) = 1-(2-3) = 1-(-1) = 1+1 = 2$
$(a-b)-c = (1-2)-3 = (-1)-3 = -4$

   (b) (Quiz3 #2)

$$0 \times a = 0$$

Start (+.5)
model of mult (+1)

Always true (+.5)

Recall we can think of multiplying by a as stretching by a.
Zero, stretched by a is still zero.

(+1) applied

2. [4] (ProblemSolvingActivity #5) A theater is set up in such a way that there are 14 seats in the first row and 4 additional seats in each consecutive row. The last row has ~~84~~ 82 seats. How many seats are in the theater? Provide justification but you do NOT need to explain as you would to a 4th grader.

(corrections where in purple)

Start (+.5)
clear/use names (+.5)
sense (+.5)

(+.5)

(last row)
2
1

82 chairs →
14+4 chairs →
14 chairs →

We need to add up the total number of chairs

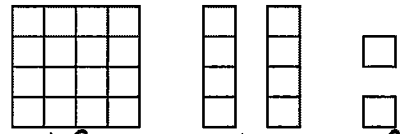
(+1)
Chairs in row 1 + Chairs in Row 2 + . . . + 82
$14 + (14+4) + (14+4)+4 + \dots + 82$
$14 + 18 + 22 + \dots + 74 + 78 + 82$

(+1) We could add up the 18 terms by hand or
$\underbrace{96 + 96 + \dots + 96}_{9} = 9 \times 96$
$= 864$

note
#chairs in n^{th} row $= 14 + 4(n-1)$
So last row number would be
$82 = 14 + 4(n-1)$
$-14 \quad -14$
$\frac{68}{4} = \frac{4(n-1)}{4}$
$17 = n-1$
$18 = n$

3. Consider the number in the base pieces below with 1 flat, 2 longs, and 2 units.

(a) [2] (Quiz2#2) Write the number of units in positional notation for the given base.



base of 4 (+.5)

1 flat 2 longs 2 units (+.5)

122_{four} (+1)

(b) [2] (NumberSystemActivity #3) Determine the total number of units, reporting in the Hindu-Arabic number system.

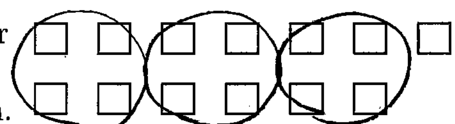
(+.5) [We need to count up the individual units with our normal system.

notation (+.5)
unpack (+.5)

$1 \times (4^2) + 2 \times (4^1) + 2 \Rightarrow 16 + 8 + 2 = \underline{26}$ (+.5)

4. (§3.1 #12) Consider the number of units shown below.

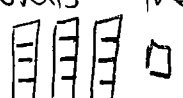
(a) [2] Sketch the minimum number of base pieces for base four to represent the set of units shown.



} 13 units

sense (+.5)

(+1)
? of flats - in base 4 flats have 4^2 or 16 units => no flats
\# of longs - we can group together sets of 4 for each long circled above => 3 longs
\# of units - the units that do not fit into a perfect long => 1

So  } (+.5)

(b) [1] Write the number of units in positional notation for base four.

(+1) 31_{four}

5. [3] Which of the Egyptian, Roman, or Babylonian number systems is most like the one most used in the United States? Provide justification for your answer.

(+.5) [The Babylonian ?

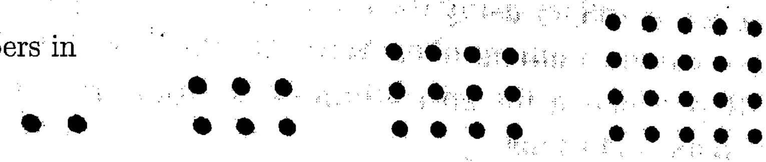
start (+.5) Neither the Egyptian nor the Roman are positional number systems
correct statements (+1) where as both the Babylonian & the US's system are.
positional sys (+.5) For example ∩ = ∩ in Egyptian but ≠
sense (+.5) and $21 \neq 12$.

Roman's number system does have position matter but not in terms of a base. Roman has a particular symbol for 1000 (M) whereas the US is using only the digits 0 thru 9.

2

6. (§1.2 #31) Consider the sequence of numbers illustrated below.

(a) [3] Find the next two numbers in the sequence.



2	6	12	20
1×2	2×3	3×4	4×5

start (+.5)
find pattern (+1)
notation (+.5)

(+.5) $\boxed{5 \times 6 \text{ or } 30}$ and (+.5) $\boxed{6 \times 7 \text{ or } 42}$

(b) [2] Identify if the sequence is recursive, arithmetic, geometric, or none of the above. Justify your answer. Could be recursive but not obviously using previous entries ....

def of arith/geom (+.5)
got it (+.5)

sequence : 2 6 12 20

(+.5)(difference +4 +6 => not arithmetic

(+.5)(factor ×3 ×2 => not geometric

I'd say none of them

(c) [2] Find the 50th number in the sequence.

start (+.5)

$50 \times 51 = \underline{2550}$ (+.5) use pattern (+1)

7. Show work and compute (you do not need to explain it to a 4th grader):

(a) [2] (Quiz3 #3) $213_{six} - 21_{six}$

notation (+.5)

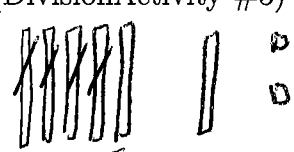
= 1 flat + 5 longs + 2 units (+.5) (+.5)

= 152_{six} — OR — 68

(+.5) broke up flat to get easy access to another long

(b) [2] (DivisionActivity #3) $112_{five} \div 4_{five}$

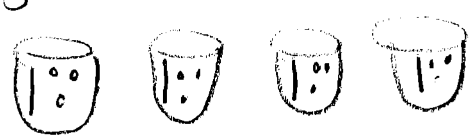
notation (+.5)
(+.5) use ÷ method share/receive/long
(1) use correctly



broke up flat to more easily divide

have 2 longs and 2 units left to share or (breaking up longs)

=> have 1 long and 3 units in each bucket



=> 13_{five} — or — 8

8. [3] Find a number that:

(+1) • is not written in base 10,
(+1) • has 2 digits, and
(+1) • is made of more than 50 units.

15_{sixty} or ᐁ ᐁᐁ

would work

9. Grade the work that follows. The work may be correct or incorrect. If correct, briefly justify why. If incorrect, find the error(s) & try to detect the reason for the error.

(a) [3] (AddActivity #1) (+.5) not correct

start (+.5)
~~long~~ sense (+.5)

$$\begin{array}{r} 42_{five} \\ +\ 34_{five} \\ \hline 76_{five} \end{array}$$

trouble (+.5)

131_{five}

(+1) [In base five we can/should regroup when we can to higher positions (much like carrying). Forgot to regroup/carry?

Eg. the 6 units can be grouped to 1 long & 1 unit
Similarly 7 longs can be grouped to 1 flat + 1 long

(b) [3] (§3.2 #25)

start (+.5)
sense (+.5)
subtraction (+.5)

(+.5) (Correct - (+1) we can maintain the difference of #'s we add the same amount to both in the subtraction

$$\begin{array}{r} 54 \\ -18 \end{array} \rightarrow \begin{array}{r} 54+2 \\ -(18+2) \end{array} \rightarrow \begin{array}{r} 56 \\ -20 \end{array} \rightarrow 36$$

(c) [3] (§3.4 #34) (+.5) not correct

start (+.5)
sense (+.5)

(+.5) trouble

$$3^2 \times 3^5 = 3^{10}$$

$$3^2 \times 3^5 = \underbrace{(3\times3)}_{2\ times} \times \underbrace{(3\times3\times3\times3\times3)}_{5\ times}$$
$$= 3^7 (= 2187)$$

(+1) [remembered the exponent rules incorrectly?

10. Consider $45 \div 3 \times 3 + 4$

(a) [1] (Quiz3 #1) Circle the operation above that should be performed first:

(b) [1] How would you modify the above expression to make it more clear the order of the operations?

$(45 \div 3) \times 3 + 4$ or $([45 \div 3] \times 3) + 4$

place brackets (+1)

11. [5] (§3.3 #6) Introduce how to multiply numbers as you would to an elementary school student who had forgotten. Use the example $123_{\text{five}} \times 4$ in the explanation.

Start +.5

interpret # (+.5)

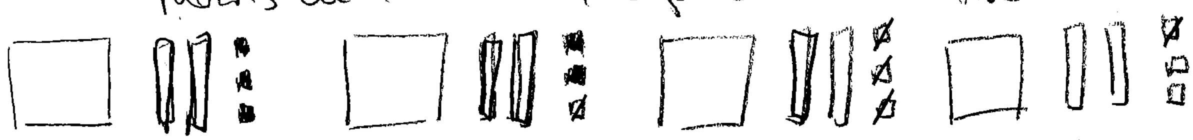
sense/notation (+1)

audience level (+.5)

Recall 123_{five} is written in positional notation & means 1 flat + 2 longs + 3 units or →

Note we are in base 5 here so our flats have 5^2 or 25 units and longs have 5 units.

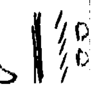
(+1) [Multiplication can be modeled with repeated addition so "×4" means we'll have 4 copies of 123_{five}. I'll draw them:



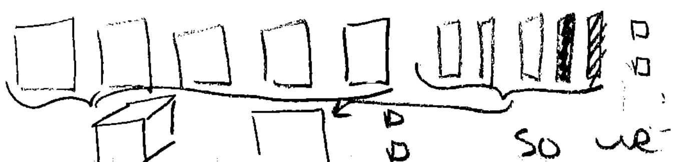
We need to only re group all of those elements so we can write our positional number. :)

regrouping (+1)

Lets start with the 12 units which we can make 2 longs + 2 units

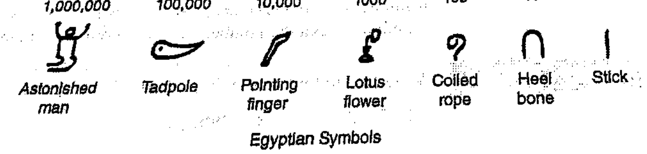


We can also find $\boxed{\text{5 longs}}$ to make 1 flat.



We have 5 flats so we can make 1 long flat. And 5 longs make 1 more flat

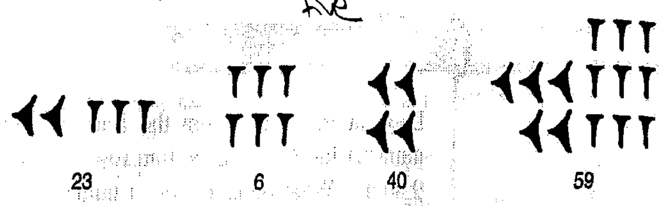
So we have 1102_{five} OR 152 (+.5)



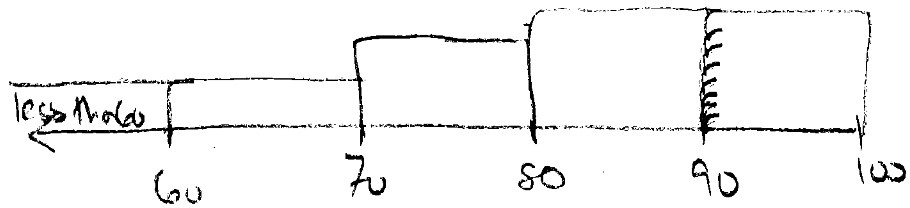
Egyptian Symbols

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

Roman Numerals



Babylonian



Median 84%

10
10
11
12
7

50