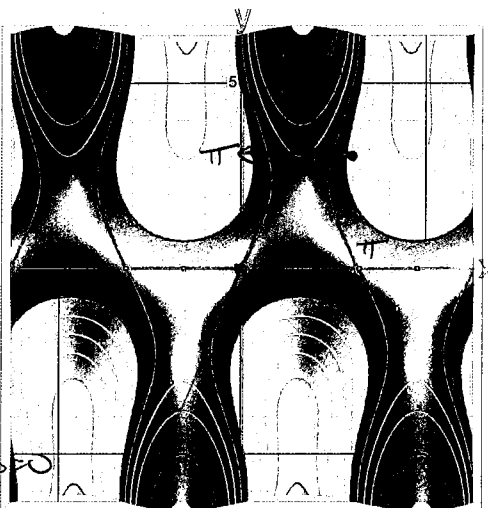


Quiz 6

Key

This is a two-stage quiz. During the first stage, use your knowledge & calculator. You have 15 min. In the second stage, you are now welcome to use your books, notes, and students in the class to retake the same quiz. You have the remainder of the quiz time to write one solution (with everyone's name on it!!!) to be turned in for the group.

1. Consider the contour map of $f(x, y) = y \sin(x) + \cos(y)$ graphed on the xy plane where $-2\pi \leq x \leq 2\pi$ and $-\pi \leq y \leq \pi$. Some multiples of $\frac{\pi}{2}$ are also plotted along the x and y axis. The contour lines are labeled by rainbow color where red is the highest and blue is the lowest.



- (a) [2] Estimate the location of two critical points of f .

$(0, 0)$ $(\pi, 0)$ (both are saddles)

(+1.5) reading coord correctly set partials to 0
 (+1) got one derivatives = 0
 (+1.5) got second

- (b) [2] Sketch the gradient vector $\nabla f(\pi, \pi)$.

$\nabla f(\pi, \pi) = \langle f_x(\pi, \pi), f_y(\pi, \pi) \rangle$ OR (more algebraically)
 vector (+1.5) $\langle -\pi, 0 \rangle$ from part (a)

- (c) [3] Find a linearization of f at (π, π) .

$z - z_0 = \frac{m_x}{x} (x - x_0) + \frac{m_y}{y} (y - y_0)$ (+1.5)

$z - \pi = -\pi(x - \pi) + 0(y - \pi)$

(+1.5) put things in right spot (+1.5)

$m_x = f_x(\pi, \pi) = y \cos(x) + 0$
 $= \pi(-1) = -\pi$

$m_y = f_y(\pi, \pi) = \sin(x) - \sin(y)$
 $= 0 - 0 = 0$

(+1.5) $[f(\pi, \pi) = \pi \sin(\pi) + \cos(\pi) = 0 + (-1)]$

2. [3] Say that we would like to find three numbers whose sum is 130 and whose product is a maximum.

- (a) Identify the function needed to be maximized. Define any variables you create.

- (b) Box any systems of equations that would need to be solved to complete this problem.

a) Looking for 3 #'s: x, y and z so that we maximize $x \cdot y \cdot z$

note $x + y + z = 130$ so we can turn this into §14.7 material.

$z = 130 - x - y$
 Maximize $P(x, y) = xy(130 - x - y)$

b) $130y - 2xy - y^2 = 0$
 $130x - x^2 - 2xy = 0$

Optimization Study #1

Workbook §4.6 #20

Tangent Study #2

Workbook §4.7 #5

derivatives (+1) get whole system

(+1)