Exam 2 # TMath 126 Winter 2024

As a reminder, you are welcome to use a two-sided 3.5" by 5" index card with notes (written or typed), a non-internet accessing calculator (which includes Desmos Test Mode) but no books, other notes, or peers.

1. [6] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

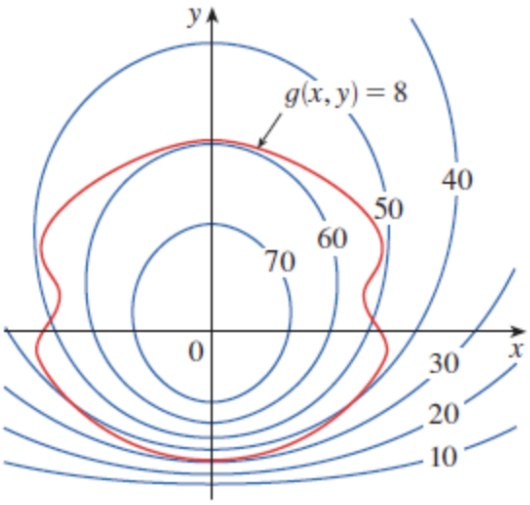
   (a) (3DActivity#3) The limit, $\lim_{(x,y)\to(2,1)} \dfrac{x-2}{x^2y-4y}$ is not defined because the limit evaluates to "0/0".

   (b) (WebHW14.6#2) Given $f(x,y) = x^2\ln(y)$ and $\overrightarrow{u} = \langle -5, 1\rangle$ we can compute the directional derivative of f in the direction of $\overrightarrow{u}$ at point $(4,1)$ as follows:

$$D_{\overrightarrow{u}} f(4,1) = \nabla f(4,1)\cdot\langle -5,1\rangle = \langle 2(4)\ln(1), 4^2\frac{1}{1}\rangle\cdot\langle -5,1\rangle = 0+16 = 16$$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [2] (WebHW14.8 #1) A contour map of f & a curve with the equation $g(x, y) = 8$ is shown. Estimate the maximum values of f subject to the constraint that $g(x, y) = 8$.



3. [6] For this problem outline (you do not actually need to find!) a solution. Make sure your outline includes:

   (a) definitions of variables used,

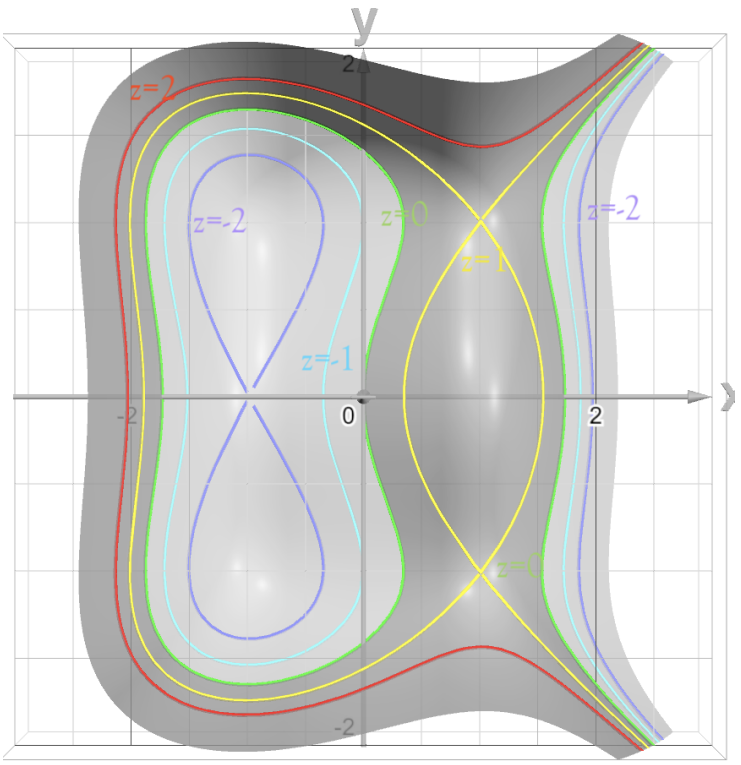
   (b) identifying the function that needs to be optimized,

   (c) boxing systems of equations that need to be solved (but do not solve them!), &

   (d) explaining how you would verify your work is correct (ie a maximum)

   (§14.7 ex6 & §14.8 ex2) A rectangular box without a lid is to be made from 12 square meters of cardboard.

4. Consider $f(x,y)$ whose contour map is shown on the right.



(a) [2] (WrittenHW14.2#46) Is $f(x,y)$ a function? Explain your reasoning.

(b) [2] (OptimizingActivity#1) Identify $(-1,0)$ as a local minimum, maximum, or saddle.

(c) [2] (WebHW14.7#2) Use the contour map to predict two extreme points for z (either local minimums or maximums).

(d) [2] (Quiz5#1) Determine if $f_x(0,0)$ is positive, zero, or negative. Explain your reasoning.

(e) [3] (Quiz6#1) *Sketch* the direction of the gradient vector $\nabla f(0,\frac{1}{2})$

(f) [3] (IntegratingActivity#1)) Estimate the signed volume trapped by $f(x,y)$, the xy plane, and above the rectangle bounded by $-1 \le x \le 0$ and $0 \le y \le 1$. Be clear with your choices so I can follow your work!

5. A function $f(x, y)$ of two variables is known to be continuous and has the values specified to the right.

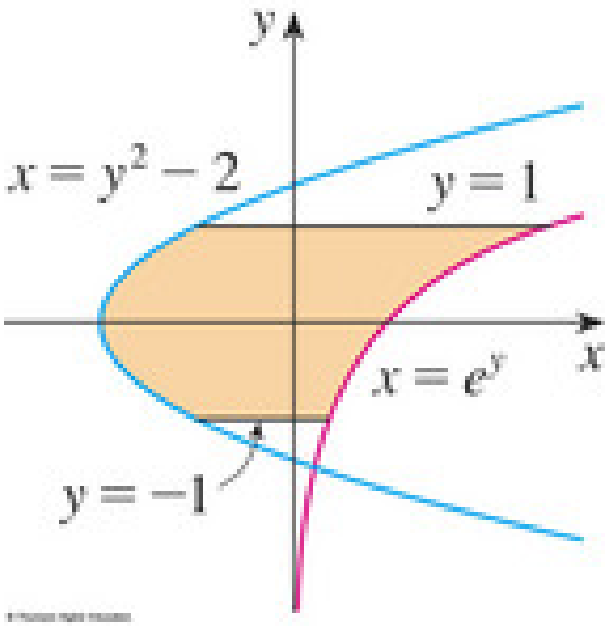
$y \setminus x$	1.0	1.1	1.2
2.0	5	7	10
2.2	4	6	8
2.4	3	5	6

   (a) [1] What is $f(1.1, 2.4)$?

   (b) [4] (PracticeExam2#3) Your boss would like you to develop a linear model that could be used to estimate the value of $f(1.4, 2.3)$. Build the model and justify the choices/steps that you make.

6. (WebHW15.2#5) Consider the volume trapped above the region shaded on the right.

   (a) [3] (WebHW14.4#3) The height function, h, is unknown. Use the fact that we know $h(-1, 0) = 1$, $h_x(-1, 0) \approx \frac{1}{2}$, and $h_y(-1, 0) \approx 1$ to find a linear approximation for h.



   (b) [4] Find an iterated integral of our linear approximation to estimate the volume. (That is, write down the expression so that technology can finish the computations.)