Median: 70
Mean: 57.0

Key

Exam 1

TMath 126

Winter 2024

1. [6] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) (WebHW12.4#2) If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$ so that $\vec{v} \times \vec{w} = 0$ (that is, the cross product of vectors v and w), then $\vec{v}$ is perpendicular to $\vec{w}$.

   Typo →

   (+.5) False

   $\vec{v} \times \vec{w}$ is a vector, 0 is a number
   vector $\neq$ number so the statement $\vec{v} \times \vec{w} = 0$
   does not make sense.

   Start (+.5)

   dot prod/cross prod (+1)
   sense (+1)

   (b) (§13.2#26) If $\vec{r}(t) = \langle t^2, \ln(et), t^3 - 3t \rangle$, then the line tangent to $\vec{r}(1)$ is:

   $$\langle x, y, z \rangle = \langle 1, 1, -2 \rangle + \langle 2t, \frac{e}{t}, 3t^2 - 3 \rangle$$

   Typo →

   (+.5) False

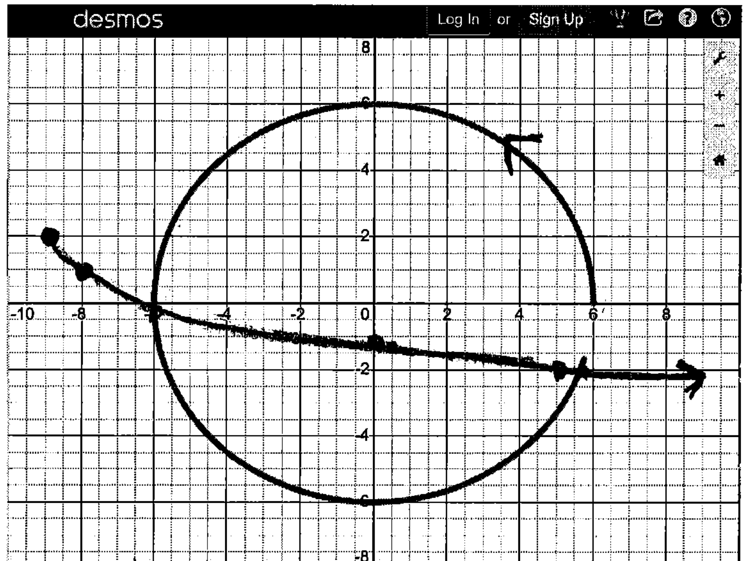
   Recall a line is of the form
   $\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle v_1, v_2, v_3 \rangle$
   where $\langle v_1, v_2, v_3 \rangle$ is a directional vector.
   That is v_1, v_2 and v_3 should be numbers
   not functions of t?

   Start (+.5)

   line def (+1)
   sense (+1)

Kin Kin

3. A plane's position is traced by a parameterized curve: $x_p(t) = t^2 - 9$ and $y_p(t) = 2 - t$ (in meters). Similarly, parameterized curves for a helicopter's position is $x_h(t) = 6\cos(t)$ and $y_h(t) = 6\sin(t)$ (in meters). The helicopter's path is traced below for $t = 0$ to 10.



(a) [1] (WebHw13.1#1) As t increases, indicate the direction of the helicopter's path by adding an arrow to the path graphed.

$X_h(0) = 6 \quad y_h(0) = 0$

(b) [2] (ParametricActivity#1) Sketch the path of the plane from $t = 0$ to $t = 8$.

shape (+1)
plug in points (+.5)
graph a bit (+.5)

(c) [4] (WrittenHW10.2 #56) Set up the expression that will return the distance traveled by the helicopter between $(6, 0)$ and $(5.6568, -2)$. Make sure your answer can be completed with technology, you do *not* need to find the numeric answer!

$\int_{\text{start point}}^{\text{end point}} \sqrt{(-6\sin(t))^2 + ((6\cos(t))^2} \, dt$

derivatives dx/dt and dy/dt (+1)
integral for arc length & used right (+1)

need to find t values

(+.5) start point ⇒ $t = 0$

(+1) end point find t so
$(5.6568, -2) = (6\cos t, 6\sin t)$
$5.6568 = 6\cos t$ | $-2 = 6\sin(t)$
⇒ $t = .34$ or 5.943 | $t = 3.481$ or 5.943

So $\int_0^{5.943} \sqrt{36\sin^2(t) + 36\cos^2 t} \, dt$
$= 35.658$ meter (+.5)
(Desmos)

(d) [3] (WordProblem #7) Find the coordinates of any points where the two paths intersect.

(+1) for technology to work I need rectangular coordinates
Plane: $t = 2 - y$
$\hookrightarrow x = (2-y)^2 - 9$ (+1)
helicopter
$x^2 + y^2 = 36$ (+.5)
(Desmos) (+.5)
@ $(5.712, -1.834)$ and $(-5.991, 0.266)$

(e) [2] (WordProblem #7) Does the plane ever collide with the helicopter? Provide justification for your answer.

(+.5) We need to find t values that plane & helicopter hit 2 points

(+1) helicopter
$(5.712, -1.836) = (6\cos(t), 6\sin t)$
⇒ $t = .311$ or 5.972
$t = 3.415$ or 5.972

plane
$(5.712, -1.836) = (t^2 - 9, 2 - t)$
$t = 3.836$
different points in time
So No (+.5)

Similarly for the 2nd point plane @ $t = 1.734$ and heli @ $t = 3.097$