Median: 70
Mean: 70

Key

Exam 1 TMath 126 Winter 2024

1. [6] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

Typo →

(a) (WebHW12.4#2) If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$ so that $\vec{v} \times \vec{w} = 0$ (that is, the cross product of vectors v and w), then $\vec{v}$ is perpendicular to $\vec{w}$.

+.5 False

$\vec{V} \times \vec{w}$ is a vector, 0 is a number
vector ≠ number so the statement $\vec{V} \times \vec{w} = 0$
does not make sense.

Start +.5

dot prod/cross prod +1
Sense +1

(b) (§13.2#26) If $\vec{r}(t) = \langle t^2, \ln(et), t^3 - 3t \rangle$, then the line tangent to $\vec{r}(1)$ is:

$$\langle x, y, z \rangle = \langle 1, 1, -2 \rangle + \langle 2t, \frac{e}{t}, 3t^2 - 3 \rangle$$

Typo →

+.5 False

Recall a line is of the form
$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle v_1, v_2, v_3 \rangle$
where $\langle v_1, v_2, v_3 \rangle$ is a directional vector.
That is v_1, v_2 and v_3 should be <u>numbers</u>
not functions of t?

Start +.5

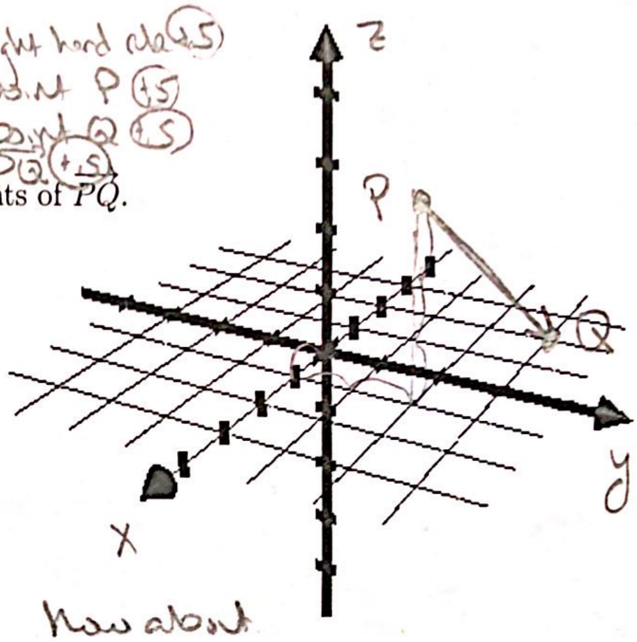
line def +1
sense +1

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the points $P(1, 2, 3)$ and $Q(-2, 3, 0)$.
Let $\vec{v} = \langle 0, -2, 1 \rangle$.

right hand rule (+.5)
point P (+.5)
point Q (+.5)
$\vec{PQ}$ (+.5)

(a) [2] (Quiz2#2) Label the x, y, and z axis and then plot the vector $\vec{PQ}$



(b) [1] (PracticeExam1#2) Find the components of $\vec{PQ}$.

$\langle -2-1, 3-2, 0-3 \rangle$

$\langle -3, 1, -3 \rangle$

subtraction (+.5) got it (+.5)

(c) [1] (DotActivity#2)
Find a vector parallel to $\vec{PQ}$.

(+.5) [We can scale $\vec{PQ}$ by any non zero number... How about

$2\langle -3, 1, -3 \rangle$ so $\langle -6, 2, -6 \rangle$ notation (+.5)

(d) [3] (WebHW12.3#6) Find the angle $\vec{PQ}$ makes with $\vec{v}$.

(+.5) Recall $\vec{PQ} \cdot \vec{v} = \|\vec{PQ}\| \|\vec{v}\| \cos\theta$ with $\vec{PQ}$ θ $\vec{v}$

(+1) { $\Rightarrow \langle -3, 1, -3 \rangle \cdot \langle 0, -2, 1 \rangle = \sqrt{9+1+9}\sqrt{0+4+1}\cos\theta$ (+.5)

$\Rightarrow 0 - 2 - 3 = \sqrt{19}\sqrt{5}\cos\theta$

notation (+.5)
solve for θ (+.5)

$\Rightarrow \dfrac{-5}{\sqrt{19}\sqrt{5}} = \cos\theta$ So $\theta = \arccos\left(\dfrac{-5}{\sqrt{19}\sqrt{5}}\right) \approx$ 2.109 rad or 120.9°

(e) [3] (WebHW12.5 #4) Find an equation of a plane passing through $(0, -2, 1)$ and normal/orthogonal/perpendicular to $\vec{v}$

start (+.5)
eq of plane (+1)
plug in (+1)
got it (+.5)

We can use $\vec{n} \cdot (\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle) = 0$

So $\langle 0, -2, 1 \rangle \cdot (\langle x, y, z \rangle - \langle 0, -2, 1 \rangle) = 0$

OR

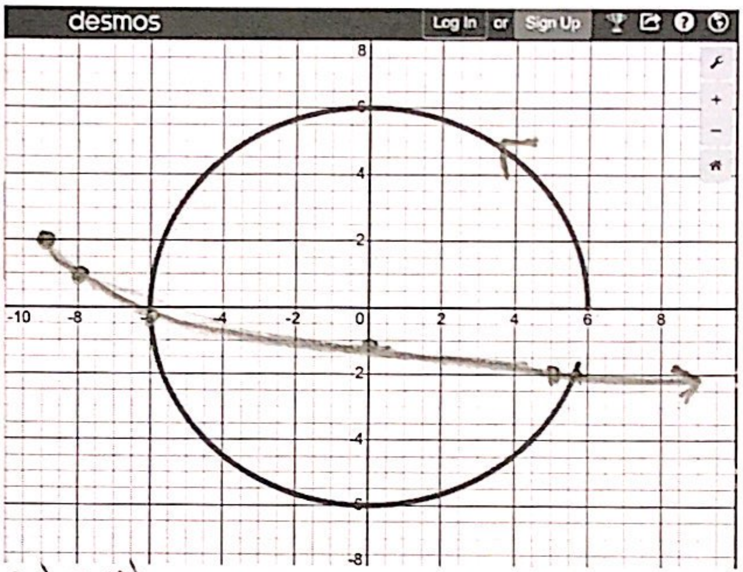
$v_1(x - x_0) + v_2(y - y_0) + v_3(z - z_0) = 0$

$0(x-0) - 2(y+2) + 1(z-1) = 0$

OR

Kin Kn

3. A plane's position is traced by a parameterized curve: $x_p(t) = t^2 - 9$ and $y_p(t) = 2 - t$ (in meters). Similarly, parameterized curves for a helicopter's position is $x_h(t) = 6\cos(t)$ and $y_h(t) = 6\sin(t)$ (in meters). The helicopter's path is traced below for $t = 0$ to 10.



(a) [1] (WebHw13.1#1) As t increases, indicate the direction of the helicopter's path by adding an arrow to the path graphed.

$x_h(0) = 6 \quad y_h(0) = 0$

(b) [2] (ParametricActivity#1) Sketch the path of the plane from $t = 0$ to $t = 8$.

Plug in points (+.5) shape (+1)
find a bit (+.5)

(c) [4] (WrittenHW10.2 #56) Set up the expression that will return the distance traveled by the helicopter between $(6, 0)$ and $(5.6568, -2)$.
Make sure your answer can be completed with technology, you do *not* need to find the numeric answer!

$$\int_{\text{start point}}^{\text{end point}} \sqrt{(-6\sin(t))^2 + ((6\cos(t))^2}\, dt$$

derivatives dx/dt and dy/dt (+1)
integral for arc length & used right (+1)

need to find t values

(+.5) start point $\Rightarrow t = 0$
(+1) end point find t so
$(5.6568, -2) = (6\cos t, 6\sin t)$
$5.6568 = 6\cos t \quad | \quad -2 = 6\sin(t)$
$\Rightarrow t = \cancel{.34}$ or $5.943 \quad | \quad \cancel{t = 3.481}$ or 5.943

So $\int_0^{5.943} \sqrt{36\sin^2(t) + 36\cos^2 t}\, dt$
$= 35.658$ meters (+.5)
(Desmos)

(d) [3] (WordProblem #7) Find the coordinates of any points where the two paths intersect.

(+1) for technology to work I need rectangular coordinants
Plane: $t = 2 - y$
$\hookrightarrow x = (2-y)^2 - 9$ (+1)
helicopter
$x^2 + y^2 = 36$ (+.5)
(Desmos) (+.5)
@ $(5.712, -1.836)$ and $(-5.994, 0.266)$

(e) [2] (WordProblem #7) Does the plane ever collide with the helicopter? Provide justification for your answer.

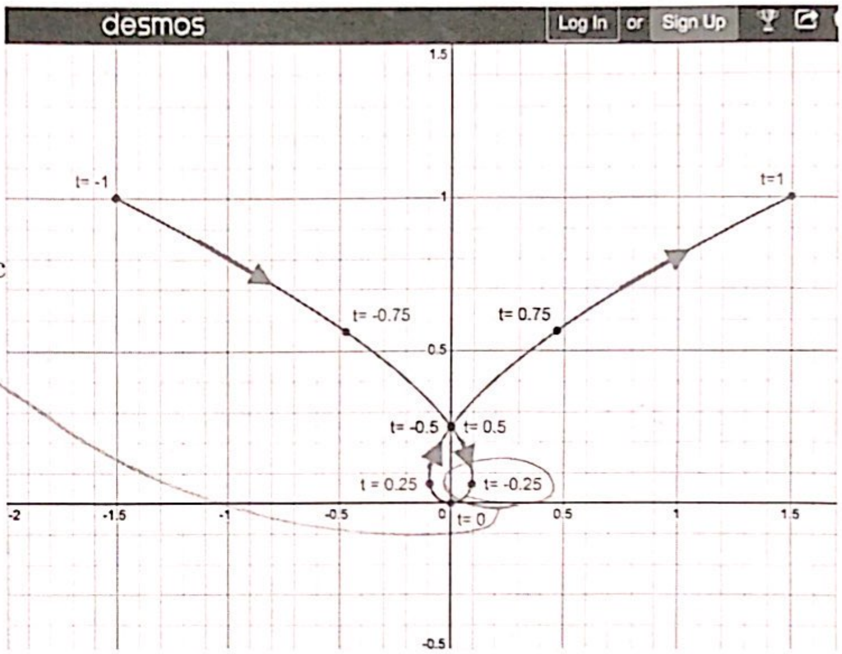
(+.5) We need to find t values that plane & helecopter hit 2 points

(+1) helecopter
$(5.712, -1.836) = (6\cos(t), 6\sin t)$
$\Rightarrow t = \cancel{.311}$ or 5.972
$t = \cancel{3.}415$ or 5.972

plane
$(5.712, -1.836) = (t^2 - 9, 2 - t)$
$t = 3.836$
different points in time
So No (+.5)

Similarly for the 2nd point
plane @ $t = 1.734$
and heli @ $t = 3.01$

4. Consider the parametric curve $x = f(t)$, $y = g(t)$ where $-1 \le t \le 1$, graphed below for the following questions.



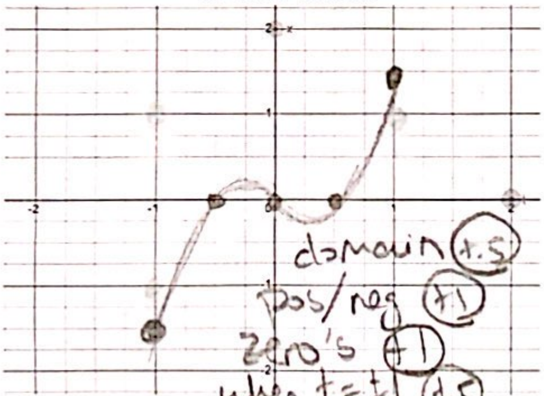
(a) [1] Identify/Estimate the point on the parametric curve when $t = -0.25$.

$\approx (.1, .05)$

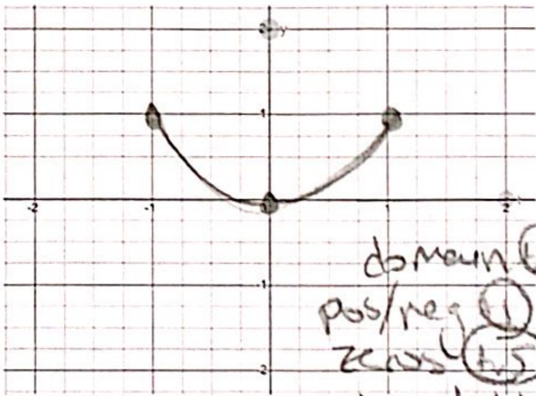
(b) [1] Identify/Estimate a point on the plane that the path passes through more than once.

$(0, .25)$

(c) [6] (WrittenHW§10.1#32) Sketch the equations $x = f(t)$ and $y = g(t)$ on the pair of axis below.



domain (+.5)
pos/neg (+1)
zero's (+1)
when $t=\pm1$ (+.5)



domain (+.5)
pos/neg (+1)
zeros (+.5)
when $t=\pm1$ (+1)

(d) [4] (WebHW10.2#3) Given the following information, find the line tangent to the curve $x = f(t)$, $y = g(t)$ when $t = \frac{1}{2}$. Use whatever form of a line you like (eg. parametric, slope-intercept, standard, etc)

$\overbrace{f\left(\frac{1}{2}\right) = 0}^{x}$ $\quad \overbrace{g\left(\frac{1}{2}\right) = .23}^{y}$ $\quad \frac{df}{dt}\left(\frac{1}{2}\right) = 2$ $\quad \frac{dg}{dt}\left(\frac{1}{2}\right) = -3.68$

(+.5) looking for line in 2D: $y - y_0 = m(x - x_0)$ OR (+.5) looking for line $\langle x, y\rangle = \langle x_0, y_0\rangle + t\vec{v}$

m = slope of line tangent to graph @ $t = \frac{1}{2}$

(+.5) $= \left.\frac{dy}{dx}\right|_{t=\frac{1}{2}}$

(+1.5) $= \dfrac{\left.\frac{dy}{dt}\right|_{t=\frac{1}{2}}}{\left.\frac{dx}{dt}\right|_{t=\frac{1}{2}}} = \dfrac{-3.68}{2} = -1.84$ 4

(+1.5) So $y - .23 = -1.84(x - 0)$

(+.5) $\vec{v}$ = directional vector of curve when $t = \frac{1}{2}$

(+1) $= \langle 2, -3.68\rangle$

(+2) So $\langle x, y\rangle = \langle 0, .23\rangle + t\langle 2, -3.68\rangle$

where t is a parameter