Exam 1 # TMath 126 Winter 2024

As a reminder, you are welcome to use a two-sided 3.5" by 5" index card with notes (written or typed), a non-internet accessing calculator (which includes Desmos Test Mode) but no books, other notes, or peers.

1. [6] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

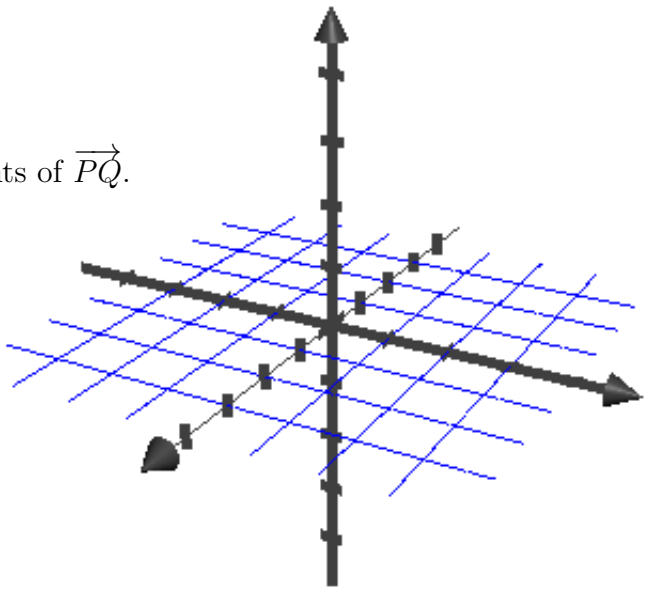
   (a) (WebHW12.4#2) If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$ so that $\vec{v} \times \vec{w} = 0$ (that is, the cross product of vectors v and w), then $\vec{v}$ is perpendicular to $\vec{w}$.

   (b) (§13.2#26) If $\vec{r}(t) = \langle t^2, \ln(et), t^3 - 3t \rangle$, then the line tangent to $\vec{r}(1)$ is:

   $$\langle x, y, z \rangle = \langle 1, 1, -2 \rangle + \langle 2t, \frac{e}{t}, 3t^2 - 3 \rangle$$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the points $P(1,2,3)$ and $Q(-2,3,0)$.
Let $\overrightarrow{v} = \langle 0,-2,1\rangle$.



(a) [2] (Quiz2#2) Label the x, y, and z axis and then plot the vector $\overrightarrow{PQ}$

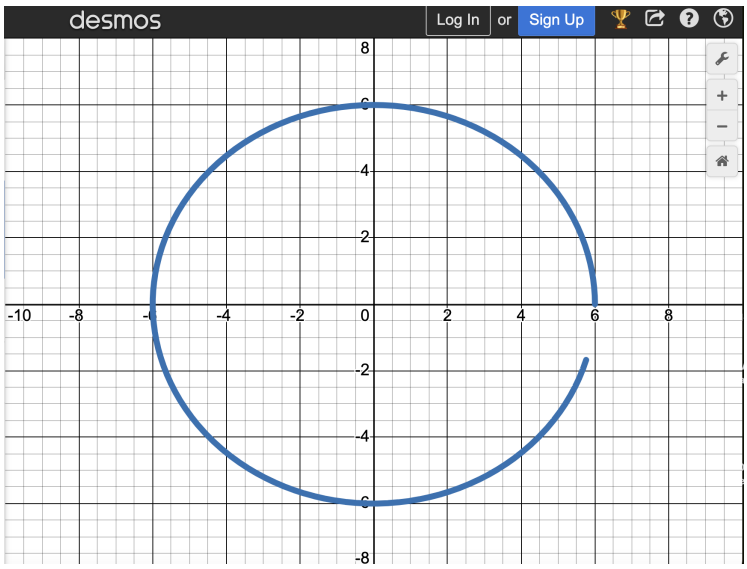
(b) [1] (PracticeExam1#2) Find the components of $\overrightarrow{PQ}$.

(c) [1] (DotActivity#2)
Find a vector parallel to $\overrightarrow{PQ}$.

(d) [3] (WebHW12.3#6) Find the angle $\overrightarrow{PQ}$ makes with $\overrightarrow{v}$.

(e) [3] (WebHW12.5 #4) Find an equation of a plane passing through $(0,-2,1)$ and normal/orthogonal/perpendicular to $\overrightarrow{v}$

3. A plane's position is traced by a parameterized curve: $x_p(t) = t^2 - 9$ and $y_p(t) = 2 - t$ (in meters). Similarly, parameterized curves for a helicopter's position is $x_h(t) = 6\cos(t)$ and $y_h(t) = 6\sin(t)$ (in meters). The helicopter's path is traced below for $t = 0$ to 10.



(a) [1] (WebHw13.1#1) As t increases, indicate the direction of the helicopter's path by adding an arrow to the path graphed.

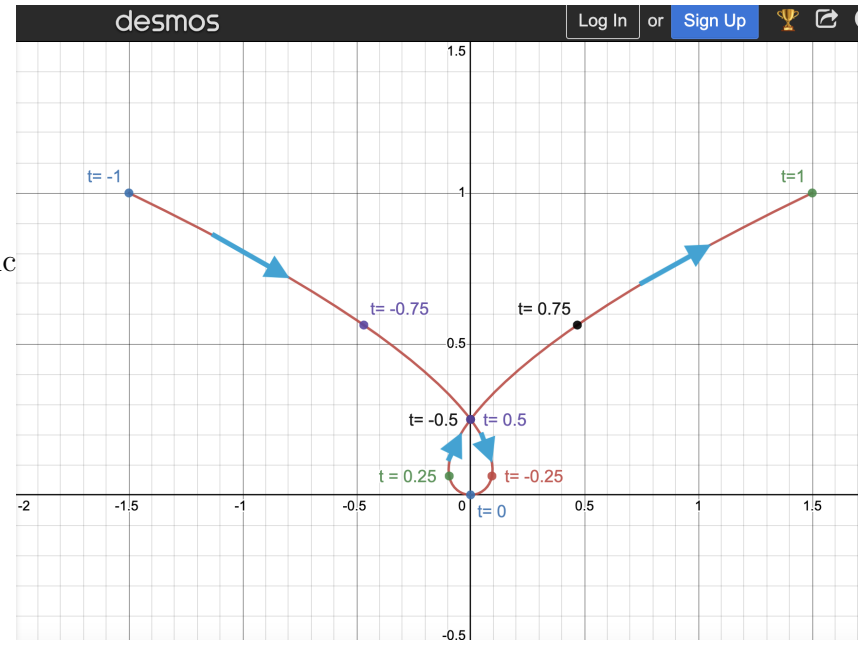
(b) [2] (ParametricActivity#1) Sketch the path of the plane from $t = 0$ to $t = 8$.

(c) [4] (WrittenHW10.2 #56) Set up the expression that will return the distance traveled by the helicopter between $(6, 0)$ and $(5.6568, -2)$. Make sure your answer can be completed with technology, you do *not* need to find the numeric answer!

(d) [3] (WordProblem #7) Find the coordinates of any points where the two paths intersect.

(e) [2] (WordProblem #7) Does the plane ever collide with the helicopter? Provide justification for your answer.

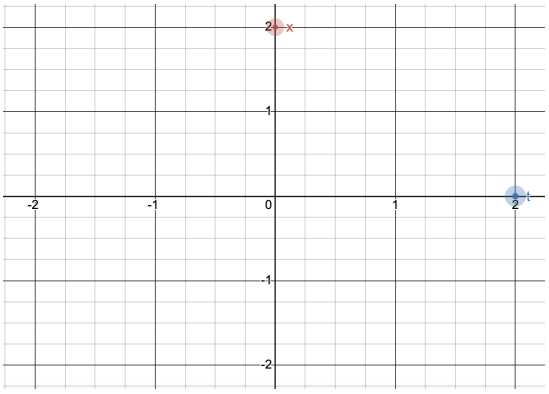
4. Consider the parametric curve $x = f(t)$, $y = g(t)$ where $-1 \leq t \leq 1$, graphed below for the following questions.

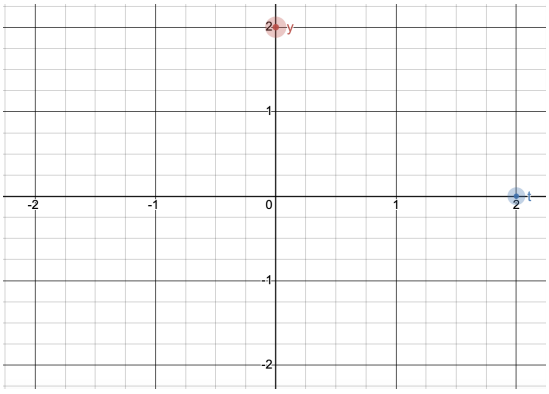


(a) [1] Identify/Estimate the point on the parametric curve when $t = -0.25$.

(b) [1] Identify/Estimate a point on the plane that the path passes through more than once.

(c) [6] (WrittenHW§10.1#32) Sketch the equations $x = f(t)$ and $y = g(t)$ on the pair of axis below.





(d) [4] (WebHW10.2#3) Given the following information, find the line tangent to the curve $x = f(t)$, $y = g(t)$ when $t = \frac{1}{2}$. Use whatever form of a line you like (eg. parametric, slope-intercept, standard, etc)

$f\left(\frac{1}{2}\right) = 0$ $\quad$ $g\left(\frac{1}{2}\right) = .23$ $\quad$ $\frac{df}{dt}\left(\frac{1}{2}\right) = 2$ $\quad$ $\frac{dg}{dt}\left(\frac{1}{2}\right) = -3.68$