Key

TMATH 126: Quiz 3

You may use:

- any kind of calculator that cannot access the internet and
- a double-sided 3 × 5" card for this quiz.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

   T **F** If $\vec{a}$ and $\vec{b}$ are vectors, then $\vec{a}$ is parallel to $\vec{b}$ if and only if $\vec{a}\cdot\vec{b} = 1$.

   +.5

   Start +.5
   Know dot + cos connection +1
   sense +.5
   complete/counter ex +.5

   Recall that $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|}$ where θ is the angle between $\vec{a}$ and $\vec{b}$. If $\vec{a}$ points in the opposite direction to $\vec{b}$ then $\theta = 180°$ and $\vec{a}\cdot\vec{b}$ could equal -1.
   Or consider $\langle 1,1,1\rangle$ and $\langle 2,2,2\rangle$. The vectors are // but the dot prod is 6.

   T F The vector $\langle 0, -1, 3\rangle$ is equivalent to $\overrightarrow{AB}$ where $A = (2, 4, 0)$ and $B = (2, 3, 3)$.

   +.5

   Start +.5
   Know vector def +1
   sense +.5
   complete +.5

   Note $\overrightarrow{AB} = \langle 2-2, 3-4, 3-0\rangle = \langle 0, -1, 3\rangle$
   So $\overrightarrow{AB}$ has the same components as $\langle 0,-1,3\rangle$.
   Since direction + magnitude are the only defining characteristics of a vector (not position) $\overrightarrow{AB} \cong \langle 0,-1,3\rangle$

2. [1] Find the length of the vector $\vec{i}$

   $\underbrace{\vec{i} = \langle 1,0,0\rangle}_{+.5}$ so $\|\vec{i}\| = \sqrt{1^2+0^2+0^2} = \underbrace{1}_{+.5}$

3. [3] (§12.1 #27 & §12.3) Describe in words the region of $\mathbb{R}^3$ represented by the inequality. (Optionally, you can try to draw it.)

   All (x, y, z) such that $\langle x, y, z\rangle \cdot \langle 1, 0, 0\rangle = 0$.

   Start +.5
   Connect to θ +.5
   got angle right +1
   complete +.5
   sense +.5

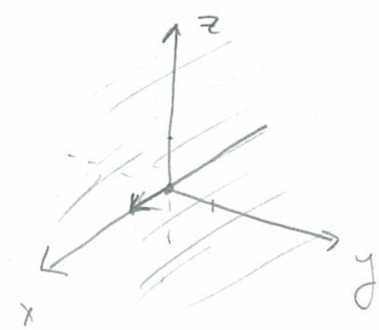
   This is all vectors in standard position that are perpendicular to the vector $\langle 1,0,0\rangle$.

   — or —

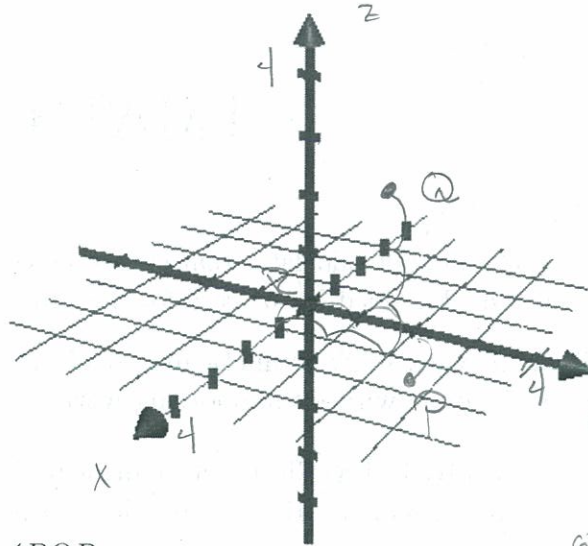
   This is the plane crossing through $(0,0,0)$ with normal vector $\langle 1,0,0\rangle$.

   — or —

   This is the yz plane



4. Let $P = (0, 2, -1)$, $Q = (1, 2, 3)$ and $R = (0, 0, 0)$.



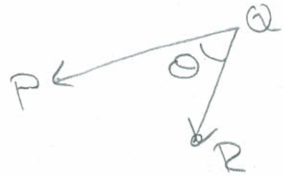
(a) [2] (Vector Wks #2) Label your positve x, y, and z axis and plot P, Q and R.

obey right hand rule (+1)
Plot P (+.5)
Plot Q (+.5)

(b) [4] (WebHW8 #6) Find the angle of $\angle PQR$

$\overrightarrow{QP} = \langle 0-1, 2-2, \cancel{-1-3} \rangle$ (1-3) $= \langle -1, 0, -4 \rangle$ (+1)

$\overrightarrow{QR} = \langle 0-1, 0-2, 0-3 \rangle = \langle -1, -2, -3 \rangle$



(+1)

Recall $\cos\theta = \dfrac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{\|\overrightarrow{QP}\| \, \|\overrightarrow{QR}\|} = \dfrac{\langle -1, 0, -4 \rangle \cdot \langle -1, -2, -3 \rangle}{\sqrt{1+0+16} \cdot \sqrt{1+4+9}}$

(+1) $= \dfrac{1+0+12}{\sqrt{17} \cdot \sqrt{14}} = \dfrac{+13}{7\sqrt{2}}$ (+1) algebra/computation

So $\theta = \arccos\left(\dfrac{+13}{7\sqrt{2}}\right) \approx 32.6°$

(c) [4] (WebHW8 #14) Find a nonzero vector orthogonal (perpendicular) to the plane passing through P, Q, and R.

note that the cross product of $\overrightarrow{QP}$ and $\overrightarrow{QR}$ would work.

using vectors (+1)
cross product (+1.5)
1) computation/alg sense (+.5)

$$\begin{vmatrix} i & j & k \\ -1 & 0 & -4 \\ -1 & -2 & -3 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & -4 \\ -2 & -3 \end{vmatrix} - \vec{j}\begin{vmatrix} -1 & -4 \\ -1 & -3 \end{vmatrix} + \vec{k}\begin{vmatrix} -1 & 0 \\ -1 & -2 \end{vmatrix}$$

$$= \vec{i}(0-8) - \vec{j}(3-4) + \vec{k}(2-0)$$

$$= -8\vec{i} + \vec{j} + 2\vec{k}$$

So $\langle -8, 1, 2 \rangle$ works although there are many other possible/correct answers!