Key

TMATH 126: Quiz 2

You may use:

- any kind of calculator that cannot access the internet and
- a double-sided 3 × 5" card for this quiz.

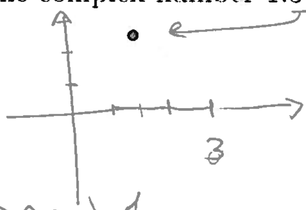
Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

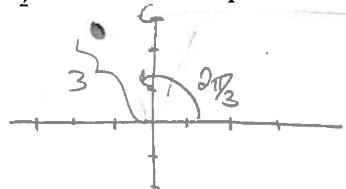
1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

sense +1
complex # rect +.5
complex # polar +.5
converted +.5

T F The complex number $1.5 + i\frac{3\sqrt{3}}{2}$ is written in polar form as $3e^{i\frac{2\pi}{3}}$

+.5





in quad 1 in quad 2

T F If the sequence a_n converges to zero, then Σa_n converges.

+.5

Consider the harmonic series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$

sense +1
notation +.5
reasoning +1

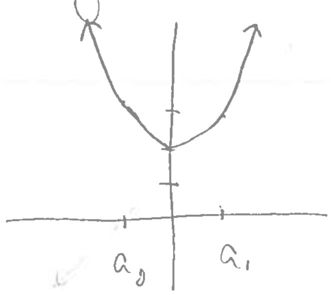
note $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$ but $\sum_{n=1}^{\infty} \frac{1}{n}$ does not converge.

explain +1.5
start +.5
function +1
initial value +.5
notation +.5

2. [4] Create an example function (either algebraically or graphically) and choose an initial value such that Newton's method will fail to converge to the closest root. Explain precisely how Newton's method will fail.

the function might not have a root:

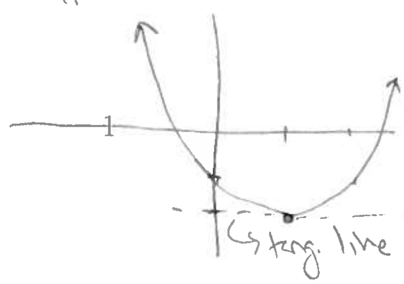
$y = x^2 + 2$
let
$a_1 = 1$



or

you may run into a tangent line whose slope is zero
=> there exists no x-intercept to find the next term

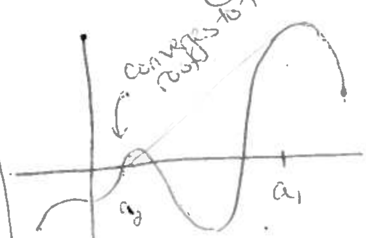
$y = (x-1)^2 - 2$
$a_1 = 1$



↳ tang. line w/ no intercept

or

you might choose an initial value that converges to a root further away



notation (+.5)
start (+.5)
geometric series (+.5)
convergence formula (+1)
alg (+1)
give series as answer (+1)
choose a or r (+.5)

3. [5] Create a *series* that will converge to 3.
Hint: we only know one type of series whose limit can be computed with a formula...

Pick a and r so that $a + ar + ar^2 + ar^3 + \ldots = 3$

or $\frac{a}{1-r} = 3$

Try letting $r = 2/3$ so $\frac{a}{1-2/3} = 3 \Rightarrow \frac{a}{1/3} = 3 \Rightarrow a = 1$

So $1 + 2/3 + (2/3)^2 + (2/3)^3 + (2/3)^4 + \ldots$

Note: there are MANY series that can be shown to converge to 3

4. (WebHW4 #9) The nth partial sum of a series $\lim_{n\to\infty} a_n$ is $s_n = \dfrac{2n-1}{3n+1}$.

(a) [2] Find $\lim_{n\to\infty} a_n$

0

(+.5)

start (+.5)

Since $\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{2n-1}{3n+1} = 2/3$ partial +.5

the partial sum converging implies we must be adding up smaller + smaller (ie closer to zero) terms.

or if you did (b) first

(b) [3] Find a formula for a_n.

$a_1 + a_2 + a_3 + \ldots + a_n = s_n$ (+.5)

$a_n = s_n - (a_1 + a_2 + \ldots + a_{n-1})$ (+.5)

$= s_n - s_{n-1}$ (+.5)

$= \overbrace{\left(\frac{2n-1}{3n+1}\right)}^{+.5} - \overbrace{\left(\frac{2(n-1)-1}{3(n-1)+1}\right)}^{+1}$

or $= \frac{2n-1}{3n+1} - \frac{2n-3}{3n-2}$

or $= \frac{(2n-1)(3n-2) - (2n-3)(3n+1)}{(3n+1)(3n-2)}$

$= \frac{\cancel{6n^2} - 4n - 3n + 2 - \cancel{6n^2} - 2n + 9n + 3}{(3n+1)(3n-2)}$

or

alternative to (a)

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{-4n-3n+2-2n+9n+3}{(3n+1)(3n-2)}$

$= \lim_{n\to\infty} \frac{5}{(3n+1)(3n-2)}$

$= 0$ by "$\frac{1}{Big}$ = Little"

0