Key

TMATH 126: Quiz 1

You may use:

- any kind of calculator that cannot access the internet and
- a double-sided 3×5" card for this quiz.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

T F Sequences are a list of numbers.

idea of def (+.5) sense (+.5) start exploring (+.5)

(+.5) order matters? (+1)
unless in your definition of 'list' there is an order...
ex $\{1,2,3,4,\ldots\} \neq \{2,1,4,3,\ldots\}$

T F The recursive sequence $a_n = -a_{n-1}$ diverges no mater the choice of a_1.

idea of notation (+.5) sense (+.5) start exploring (+.5)

(+.5) Let $a_1 = 0$ then the sequence is $\{0,-0,0,-0,0,-0,\ldots\}$ (+1)
or $\{0,0,0,0,\ldots\}$ which converges to zero

2. Consider the sequence: $\left\{\frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \frac{9}{16}\cdots\right\}$. or

(a) (WebHW #3) [3] Find a formula for the n^{th} term where a_1 is the first term.

start (+.5) sense (+.5)

$a_n = \frac{2n+1}{2^n}$ (+1) (+1) works

or (didn't see a nice recursive definition...)

(b) (§11.1 #30) [2] Find the limit of the terms in the above sequence as $n \to \infty$, if it exists. Justify your work!

sense (+.5)

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2n+1}{2^n} \overset{L'H}{=} \lim_{n\to\infty} \frac{2}{2^n \ln 2} = 0$ (+1) (+.5)

b/c "$\frac{\infty}{\infty}$" $\frac{1}{\text{Big}}$

or

def sequence (+.5)
notation (+.5)
diverges (+1)

3. [2] Create a sequence that does not converge.

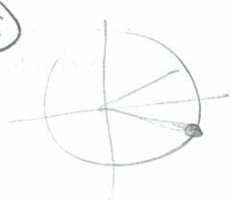
$\{1, -1, 1, -1, 1, \ldots\}$ or $a_n = (-1)^{n+1}$

4. [3] (WebHW2 #8) Determine if the following sequences converge or diverge. If it converges, find the limit.

$$a_n = \tan\left(\frac{2\pi n}{7 - 12n}\right)$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \tan\left(\frac{2\pi n}{7-12n}\right)$$

Sense (+.5)



(+.5)

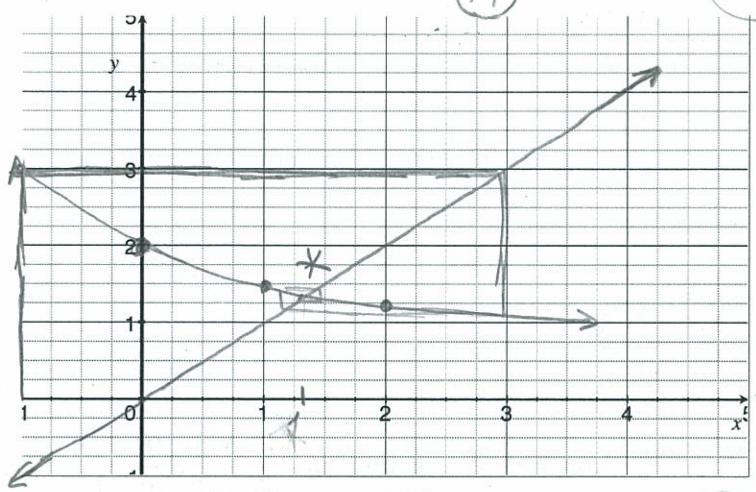
$$= \tan\left(\lim_{n\to\infty} \frac{2\pi n}{7-12n}\right)$$ b/c tan is cont @ $\pi/6$

$$= \tan\left(\lim_{n\to\infty} \frac{2\pi}{-12}\right) = \tan\left(-\frac{\pi}{6}\right) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

(+.5) (+1)

(+.5)

5. (Summer '11 Quiz 1#4)
Consider the recursively defined sequence $a_n = \frac{1}{2}^{a_{n-1}} + 1$.



(a) [1] If $a_1 = -1$, write down the first three terms of the sequence.

(+.5)

$\{-1, \left(\frac{1}{2}\right)^{-1} + 1, \left(\frac{1}{2}\right)^{3} + 1, \ldots\}$

$\{-1, 3, 9/8, \ldots\}$

(+.5)

(b) [2] If $a_1 = -1$, does the sequence converge?
If the sequence does converge, identify the limit on the graph.

$R(x) = \left(\frac{1}{2}\right)^x + 1$ (+.5)

Cobweb (+1)
got it (+.5)

Yes it converges to the x (or y) coordinate where the line x=y crosses the curve $R(x) = \left(\frac{1}{2}\right)^x + 1$ denoted by a ✱

(c) [1] What values can a_1 be to guarantee that the sequence a_n will converge?

Any x-value will work :)