Final

Tmath 126

Practice

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges to a finite number.

   (b) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that the n^{th} partial sum of a series is $s_n = \dfrac{n+5n^2}{n^2-e}$. Then $\lim_{n\to\infty} a_n = 5$.

   (c) Newton's method will approach a root of a function, if it exists, no matter the initial guess.

   (d) $(\overrightarrow{a} \times \overrightarrow{b}) \cdot \overrightarrow{a} = 0$.

(e) $||\overrightarrow{a} \times \overrightarrow{b}|| = ||\overrightarrow{b} \times \overrightarrow{a}||$.

(f) If $f(x, y)$ is a continuous function, the first-order derivatives exist, and f has a local minimum or maximum at the point $(0, 0)$, then $\nabla f(0, 0) = \overrightarrow{0}$.

(g) Let f be a function of x and y. If $\nabla f(c, d) = (2, 1)$, then the vector $\langle 2, 1\rangle$ is tangent to the contour line of the surface of f at $(c, d, f(c, d))$.

(h) $\int_{-1}^{2} \int_{0}^{6} x^2 \sin(x - y)\, dx\, dy = \int_{0}^{6} \int_{-1}^{2} x^2 \sin(x - y)\, dy\, dx$

(i) $\int_{-1}^{x} \int_{0}^{6} x^2 \sin(x - y)\, dx\, dy = \int_{0}^{6} \int_{-1}^{x} x^2 \sin(x - y)\, dy\, dx$

2. Evaluate the following if possible.

$$\lim_{n\to\infty} a_n$$

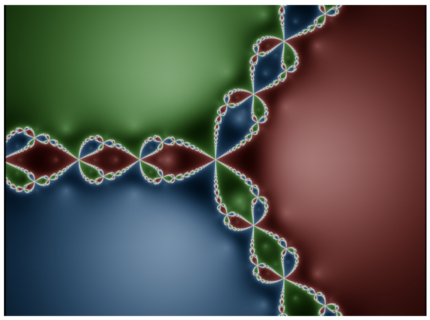
where $a_1 = 0$ and $a_{n+1} = 2^{a_n} - 3$

$$\sum_{n=0}^{\infty} \frac{n+1}{3n+2}$$

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$$

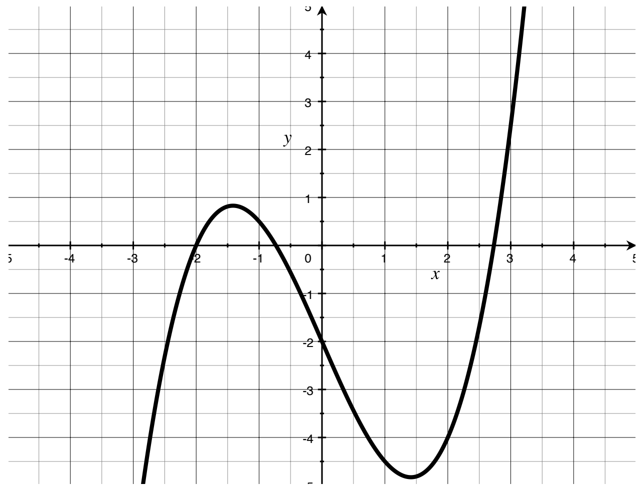
$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!}$$

3. Below is the Julia set associated with Newton's method and the polynomial $f(z) = z^3 - 1$. Identify what roots each color basin is associated with and explain how you know.



4. Find the maximum and minimum volumes of a rectangular box with the constraints that the surface area is 1500cm^2 and total edge length is 200cm.

5. The graph of $g(x) = \frac{1}{2}x^3 - 3x - 2$ is shown to the right.



   (a) Choose an initial value so that Newton's method will converge to the positive root.

   Use the graph on the right to estimate the first three approximations in Newton's method.

   (b) Find the second order Taylor polynomial $T_2(x)$ at $b = 2$.

   (c) Approximate $g(2.2)$ using $T_2(x)$.

   (d) Use Taylor's inequality to find an upper bound for the error in the approximation above.

6. (a) Find the distance between the plane $x - y + 2z = 3$ and the point (2,-1,3).

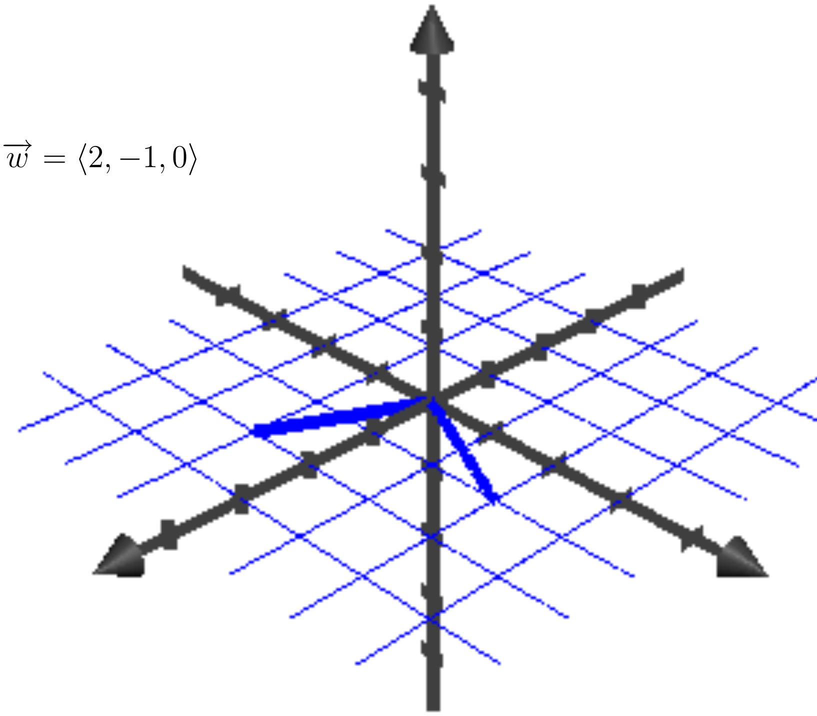
(b) Find the equation of the line of intersection between $x-y+2z = 3$ and $x+2y+3z = 0$.

Consider the vectors: $\overrightarrow{v} = \langle 1, 2, 0 \rangle$ and $\overrightarrow{w} = \langle 2, -1, 0 \rangle$

(a) [1] Draw the vector $-\overrightarrow{v}$

(b) [2] Draw the vector $\overrightarrow{w} - \overrightarrow{v}$

(c) [2] Draw the vector $\overrightarrow{v} \times \overrightarrow{w}$



8. Consider the quadratic surface given by the equation $2x^2 + 3y^2 - 5z^2 = 0$. Describe the cross sections created when sliced parallel to the xy plane. How about for those parallel to the yz plane.

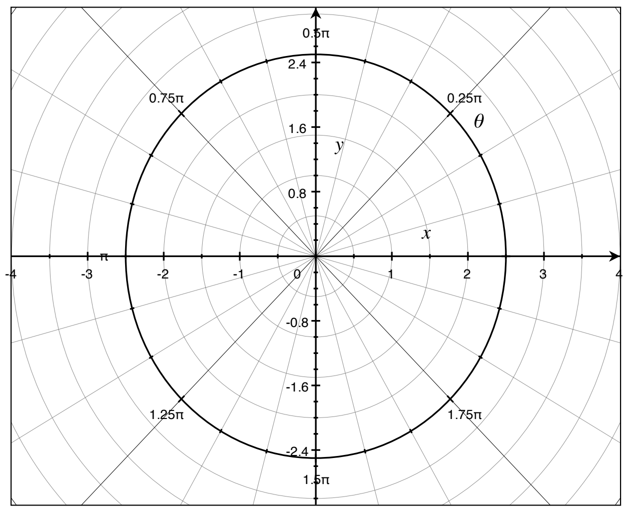
9. You are given the following data of a function $g(x, y)$. Your boss wants you to approximate $g(.8, 1.4)$ and wants to be convinced you're doing something sophisticated. Find a linear approximation for your boss and explain your choices (there are many that you will make!).

x	y	$g(x,y)$
0.55	1.2	27
0.65	1.0	31
0.65	1.1	29
0.75	1.2	50

10. Let $P(2, -\frac{2\pi}{3})$ and $Q(-3, \frac{5\pi}{6})$ be polar coordinates.

   (a) [2] Plot P and Q.

   (b) [2] Find the Cartesian coordinates of the point P.



   (c) [2] Find two other pairs of polar coordinates for the point P.

11. Consider the function $h(x, y) = x^3 - 12xy + 8y^3$.

   (a) Find all critical points of h.

   (b) Classify each critical point as a local minimum, a local maximum, or a saddle point.

12. Consider the double integral

$$\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} \cos(x)\sqrt{1+\cos^2 x}\, dx dy$$

(a) Sketch the region in the xy-plane where the integral is taken over.

(b) Switch the order of integration.

(c) Compute the double integral.