Exam 1 **Tmath 126** Summer 2014

1. [12] TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   (a) (§11.1 #84 Cobwebbing) Let $a_n$ be a recursively defined sequence where $a_1 = 4$, $a_n = f(a_{n-1})$, and $f$ is a continuous function. If $\lim_{n\to\infty} a_n = 6$, then $f(6) = 6$.

   (b) (PracticeExam #1) Given a function $f$, the associated taylor series $T$ has the property that $f(x) = T(x)$ for all $x$.

   (c) (pg778 #1) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (d) (Lecture 7/8) The first degree Taylor polynomial of a function $f$ centered at 2 is the same as the line tangent to $f$ when x=2.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the series: $\frac{1}{1} + \frac{-1}{1} + \frac{1}{2} + \frac{-1}{6} + \frac{1}{24} + \frac{-1}{120} + ...$

   (a) [3] (WebHW5 #1) Write the series using sigma notation:

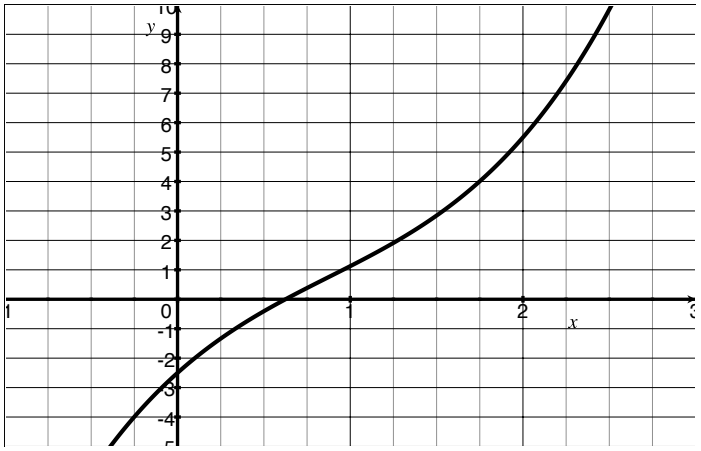
   (b) [3] (PracticeExam #4) Determine what the series above converges to, if it converges. Justify your work.

3. Compute the following if possible.

   (a) [4] (§11.1 #30) $\lim_{n\to\infty} a_n$ where $a_n = \sqrt{\frac{n+2}{25n-1}}$

   (b) [4] (WebHW4 #4) The series $\sum_{n=0}^{\infty} a_n$ where $a_n = 6(0.1)^n$

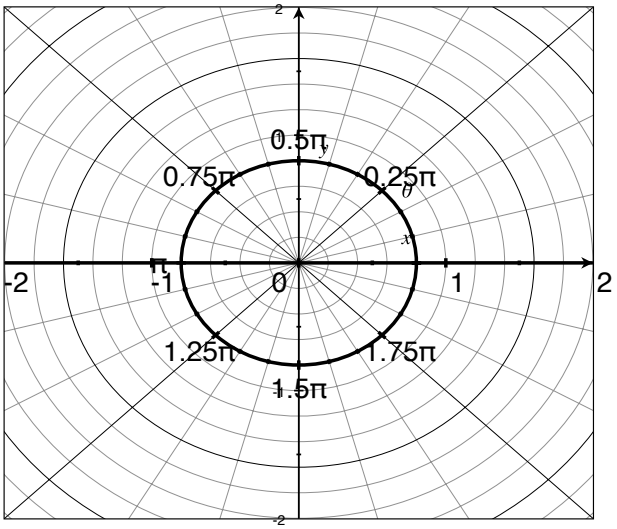
4. (PracticeExam #5)
Let $p(x) = x^3 - \frac{21}{8}x^2 + \frac{21}{4}x - \frac{5}{2}$
whose graph is shown to the right.




(a) (Quiz2 #2) [3] Will Newton's method always be able to find a root of this polynomial $p$ no mater the starting value? Justify your answer.

(b) (WebHW3 #1) [2] Choose an $x_1$ value, and identify $x_2$ using Newton's method (either numerically or graphically).

(c) [1] The complex number $R = 1 - (\sqrt{3})i$ is a root of $p$. Plot $R$ on the complex axis.




(d) [2] (ApxH #26) Convert $R$ into polar coordinates.

(e) (PracticeExam #6) [2] Compute $R^3 - 8$.

5. Consider the function $f(x) = -3\cos(x^2)$

(a) [4] (§11.10 #39) Find a power series representation for $f$. Any power series will suffice but supply work so I can see which one you are working with.

(b) [2] For what $x$ values will the power series of $x$ converge to $f(x)$?

(c) [4] Find a *reasonable* bound for the error of the second Taylor polynomial approximation centered at 0 for $f(3.5)$. Make sure that you show enough work that I know why you choose the $M$ that you did.

(d) [4] Identify a topic we covered in this class that wasn't on the exam. Create an exam question about this topic and answer it.