TMATH 126: Quiz 3

You may use:

Key

- any kind of calculator that cannot access the internet and
- a double-sided 3 × 5" card for this quiz.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

T F If $\vec{a}$ and $\vec{b}$ are vectors, then $\vec{a}$ is parallel to $\vec{b}$ if and only if $\vec{a} \cdot \vec{b} = 1$.

.5

Start +.5

/dot def +1

got it/sense +1

If $\vec{a} \parallel \vec{b}$ then the angle between them is 0 or π radians

Thus $\cos\theta$ could be 1 or -1.

So notice $\langle 1,0\rangle \parallel \langle -1,0\rangle$ but

$\langle 1,0\rangle \cdot \langle -1,0\rangle = 1\cdot -1 + 0\cdot 0 = -1$

T F The volume of a parallelepiped with edges $\overline{PQ}$, $\overline{PR}$ and $\overline{PS}$ can be found by computing $(\overrightarrow{PQ} \cdot \overrightarrow{PR}) \times \overrightarrow{PS}$.

.5

Start +.5

ex/ dot/cross def +1

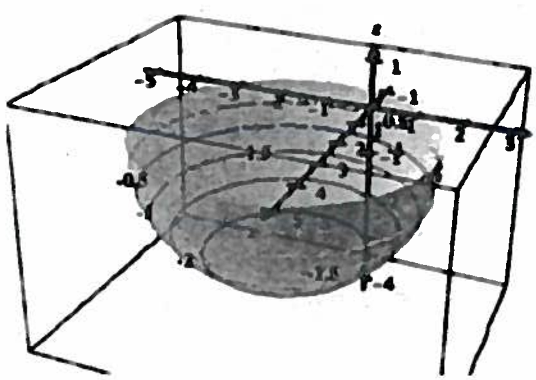
got it/sense +1

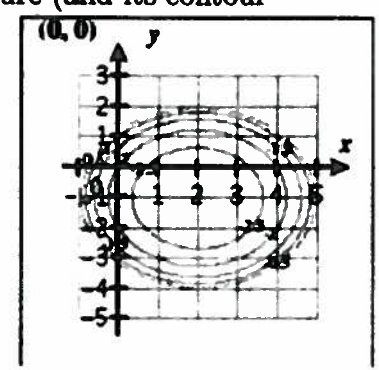
$\overrightarrow{PQ} \cdot \overrightarrow{PR}$ is a scalar/number

The cross product only acts between 2 vectors so

$(\overrightarrow{PQ} \cdot \overrightarrow{PR}) \times \overrightarrow{PS}$ makes no sense

2. [3] (§12.1 #36) Write the equation(s) for the three dimensional figure (and its contour lines) shown below.





centered at $(2,-1,0)$

radius is 3 +1

+1 eq of sphere

$r^2 = (x-h)^2 + (y-l)^2 + (z-k)^2$

$\Rightarrow 9 = (x-2)^2 + (y+1)^2 + z^2$ +1

and $z \le 0$

or $z = -\sqrt{9-(x-2)^2-(y+1)^2}$

3. Let $\vec{v} = \langle 2, -1\rangle$ and $\vec{w} = \langle 4, 3\rangle$.

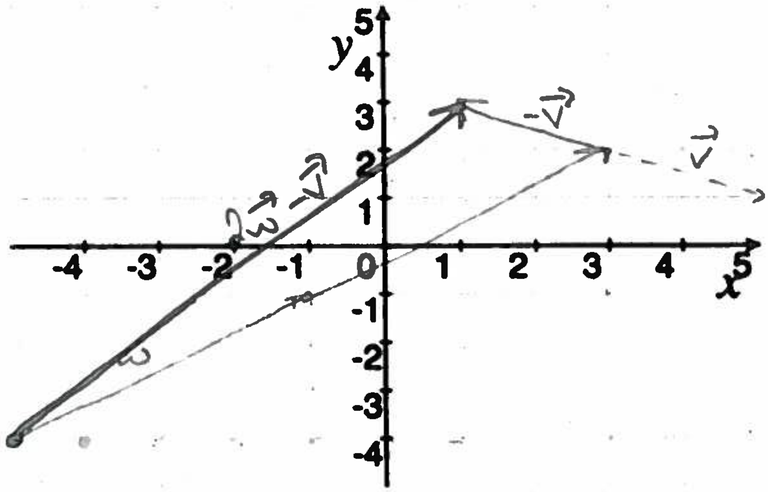
(a) [2] (WebHW7 #12) Draw and then find the components of $2\vec{w} - \vec{v}$.

$2\vec{w} - \vec{v} = 2\langle 4,3\rangle - \langle 2,-1\rangle$

$= \langle 8,6\rangle + \langle -2,1\rangle$

$= \langle 6,7\rangle$ } (+1)

vector add/sub (+1)



(b) [2] (§12.1 #8) Find $\|2\vec{w} - \vec{v}\|$ and explain what you found in terms a 7th grader would understand.

(+1) { $\|2\vec{w} - \vec{v}\| = \|\langle 6,7\rangle\| = \sqrt{6^2+7^2} = \sqrt{36+49} = \sqrt{85}$

(+1) { The length of the vector $2\vec{w} - \vec{v}$ (drawn above) is $\sqrt{85}$ or about

(c) [3] (WebHW7 #15) Find a vector parallel to $\vec{w}$ but with length 2.

$\|\vec{w}\| = \sqrt{4^2+3^2} = 5$ } (+.5)

se/plan (+1)

(+.5) { $\Rightarrow$ direction of $\vec{w}$ with unit length $\frac{1}{5}\langle 4,3\rangle = \langle \frac{4}{5}, \frac{3}{5}\rangle$

(+1) to increase the length to 2 we scale by 2

$2\langle \frac{4}{5}, \frac{3}{5}\rangle = \langle \frac{8}{5}, \frac{6}{5}\rangle$

4. [4] (WebHW8 #14) Find the are of the triangle with vertices P, Q, and R where $P(0,-2,0)$, $Q(5,1,-2)$, and $R(6,4,1)$.

(+1) { Recall $\|\vec{PQ} \times \vec{PR}\|$ is the area of a parallelogram with side lengths $\vec{PQ}$ and $\vec{PR}$ thus

Area of $\triangle PQR = \frac{1}{2}\|\vec{PQ} \times \vec{PR}\| = \underbrace{\frac{1}{2}\sqrt{15^2+17^2+12^2}}_{(+1)}$

≈

$\vec{PQ} = \langle 5-0, 1-(-2), -2-0\rangle = \langle 5,3,-2\rangle$

$\vec{PR} = \langle 6-0, 4-(-2), 1-0\rangle = \langle 6,6,1\rangle$ } (+1)

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 5 & 3 & -2 \\ 6 & 6 & 1 \end{vmatrix} = \vec{i}(3\cdot 1 + 2\cdot 6) - \vec{j}(5\cdot 1 + 2\cdot 6) + \vec{k}(5\cdot 6 - 6\cdot 3)$$

$= 15\vec{i} - 17\vec{j} + 12\vec{k}$ } (+1)