TMATH 126: Quiz 1

Key

You may use:

- any kind of calculator that cannot access the internet and
- a double-sided 3 × 5" card for this quiz.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

1) thinking
1) sense
.5) get it

T F Let $p \geq 1$, then the sequence $a_n = \left(\frac{1}{n}\right)^p$ converges.

1.5

$\lim_{n\to\infty} \left(\frac{1}{n}\right)^p = \left(\lim_{n\to\infty} \frac{1}{n}\right)^p$ by properties of limits

$= 0^p = 0$

) thinking
1) sense
1.5) found counterex

T **F** The recursive sequence $a_n = -a_{n-1}$ diverges no mater the choice of a_1.

1.5

let $a_1 = 0$ then

$\{0, -0, 0, -0, 0, -0, \ldots\} = \{0, 0, 0, \ldots\}$

which converges to zero

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2. Consider the sequence: $\left\{1, \frac{-1}{3}, \frac{1}{5}, \frac{-1}{7}, \frac{1}{9} \ldots\right\}$.

(a) (WebHW #3) [3] Find a formula for the n^{th} term where a_1 is the first term.

$\frac{1}{1}, -1\frac{1}{3}, \frac{1}{5}, -1\cdot\frac{1}{7},$ $\qquad \frac{1}{\text{odd \#}}$ $\qquad$ odd #s look like $2n \pm 1$ 1.5

$a_n = \underbrace{(-1)^{n+1}}_{+1} \underbrace{\frac{1}{2n-1}}_{+1}$ $\qquad$ notation pattern 1.5

(b) [1] Find the limit of the terms in the above sequence as $n \to \infty$.

$\underbrace{0}_{+1}$ b/c $\frac{1}{\text{Big}} \to 0$ and $\frac{-1}{\text{Big}} \to 0$

3. [5] Determine if the following sequences converge or diverge. If it converges, find the limit.

(§11.1 #27)

$a_n = e^{\frac{1}{n}}$

limit properties +1

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} e^{1/n}$$

$$= e^{\lim_{n\to\infty} 1/n}$$

+1 { note $\lim_{n\to\infty} \frac{1}{n} = 0$ b/c $\frac{1}{\text{Big}} \to 0$

+1 { so $\lim_{n\to\infty} e^{1/n} = e^0 = 1$

(Seq Wks #1)

$a_n = \frac{3^n}{2^{n-1}}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n}{2^{n-1}}$$

$$= \lim_{n\to\infty} \frac{3^n}{2^n 2^{-1}}$$

$$= \lim_{n\to\infty} 2 \cdot \left(\frac{3}{2}\right)^n$$

Diverges b/c $3/2 > 1$ } +1

algebra +.5 notation +.5

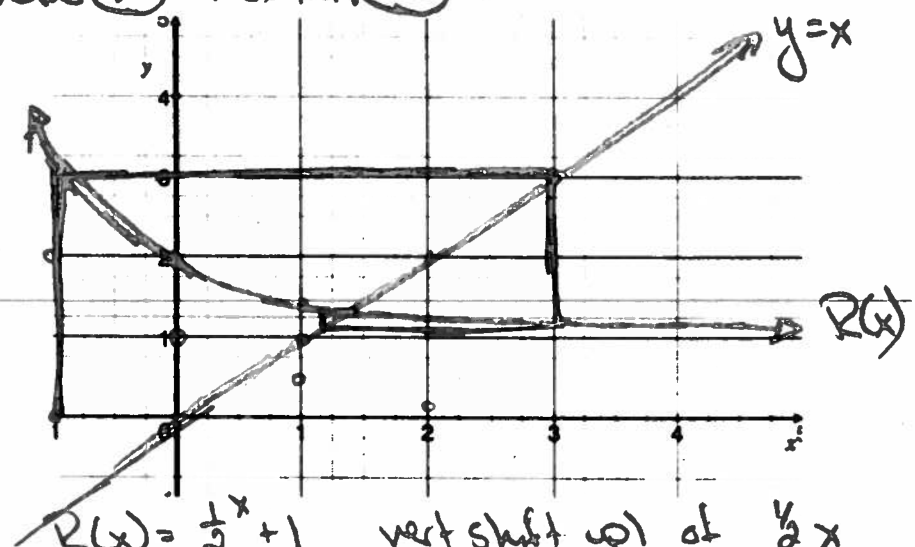
Note: I did not tell you to use cobweb

4. (Summer '11 Quiz 1#4)
Consider the recursively defined sequence $a_n = \frac{1}{2}^{a_{n-1}} + 1$.

(a) [1] If $a_1 = -1$, write down the first three terms of the sequence.

$\{-1, \frac{1}{2}^{-1} + 1, \frac{1}{2}^{3} + 1$

$\{-1, 3, \frac{1}{8} + 1, \ldots\}$ +.5 +.5



$R(x) = \frac{1}{2}^x + 1$ vert shift up 1 of $\frac{1}{2}^x$

(b) [3] If $a_1 = -1$, does the sequence converge?
If the sequence does converge, identify the limit on the graph.

cobweb:
R(x) +1
y=x +.5
process +1
graph +.5

Converges to the x coord (& y coord) of the intersection of $y=x$ & $y=R(x)$ shown above

(c) [1] What values can a_1 be to guarantee that the sequence a_n will converge?

all a_1 values will converge to the value ~~defined~~ in (b) b/c of the relative shapes of $y=x$ + $y=R(x)$