

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

Let  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  be vectors in  $\mathbb{R}^3$ .

Recall that  $\cdot$  refers to the dot product, and  $\times$  refers to the cross product.

(a) If  $\lim_{n \rightarrow \infty} a_n = 0$ , then  $\sum_{n=1}^{\infty} a_n$  converges to a finite number.

(b) Let  $\{a_n\}_{n=1}^{\infty}$  be a sequence such that the  $n^{\text{th}}$  partial sum of a series is  $s_n = \frac{n + 5n^2}{n^2 - e}$ .  
Then  $\lim_{n \rightarrow \infty} a_n = 5$ .

(c) Newton's method will approach a root of a function, if it exists, no matter the initial guess.

(d)  $(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$ .

(e)  $\|\vec{a} \times \vec{b}\| = \|\vec{b} \times \vec{a}\|.$

(f) If  $f(x, y)$  is a continuous function, the first-order derivatives exist, and  $f$  has a local minimum or maximum at the point  $(0, 0)$ , then  $\nabla f(0, 0) = \vec{0}.$

(g) Let  $f$  be a function of  $x$  and  $y$ . If  $\nabla f(c, d) = (2, 1)$ , then the vector  $\langle 2, 1 \rangle$  is tangent to the contour line of the surface of  $f$  at  $(c, d, f(c, d)).$

(h)  $\int_{-1}^2 \int_0^6 x^2 \sin(x - y) dx dy = \int_0^6 \int_{-1}^2 x^2 \sin(x - y) dy dx$

(i)  $\int_{-1}^x \int_0^6 x^2 \sin(x - y) dx dy = \int_0^6 \int_{-1}^x x^2 \sin(x - y) dy dx$

2. Evaluate the following if possible.

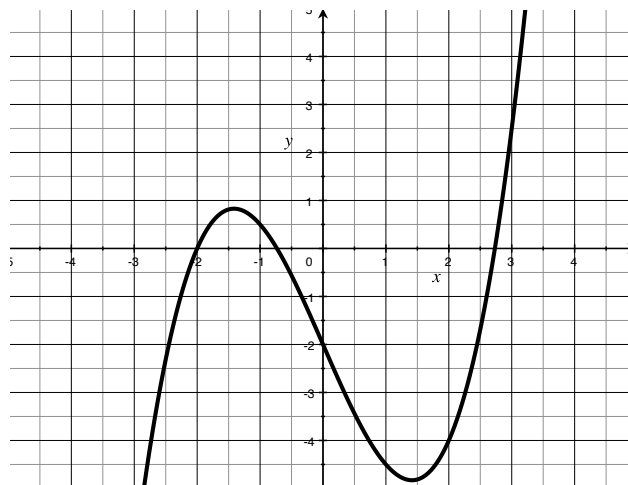
$$\lim_{n \rightarrow \infty} \sin \left( \frac{6n\pi}{5 + 8n} \right)$$

$$\sum_{n=0}^{\infty} \frac{n+1}{3n+2}$$

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!}$$

3. The graph of  $g(x) = \frac{1}{2}x^3 - 3x - 2$  is shown to the right.



- (a) Choose an initial value so that Newton's method will converge to the positive root.

Use the graph on the right to estimate the first three approximations in Newton's method.

- (b) Find the second order Taylor polynomial  $T_2(x)$  at  $b = 2$ .

- (c) Approximate  $g(2.2)$  using  $T_2(x)$ .

- (d) Use Taylor's inequality to find an upper bound for the error in the approximation above.

4. (a) Find the distance between the plane  $x - y + 2z = 3$  and the point  $(2, -1, 3)$ .

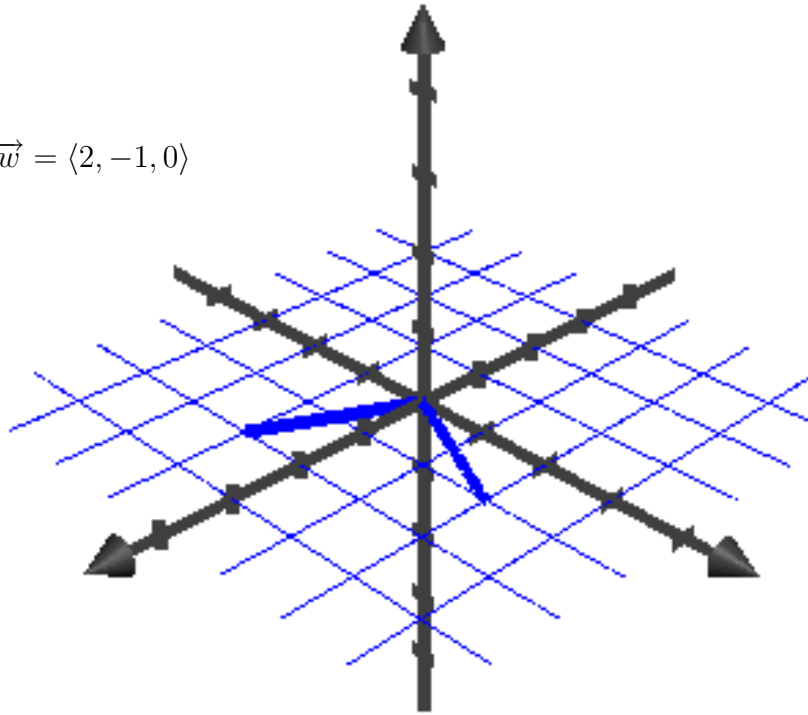
(b) Find the equation of the line of intersection between  $x - y + 2z = 3$  and  $x + 2y + 3z = 0$ .

Consider the vectors:  $\vec{v} = \langle 1, 2, 0 \rangle$  and  $\vec{w} = \langle 2, -1, 0 \rangle$

(a) [1] Draw the vector  $-\vec{v}$

(b) [2] Draw the vector  $\vec{w} - \vec{v}$

(c) [2] Draw the vector  $\vec{v} \times \vec{w}$



6. Consider the quadratic surface given by the equation  $2x^2 + 3y^2 - 5z^2 = 0$ .

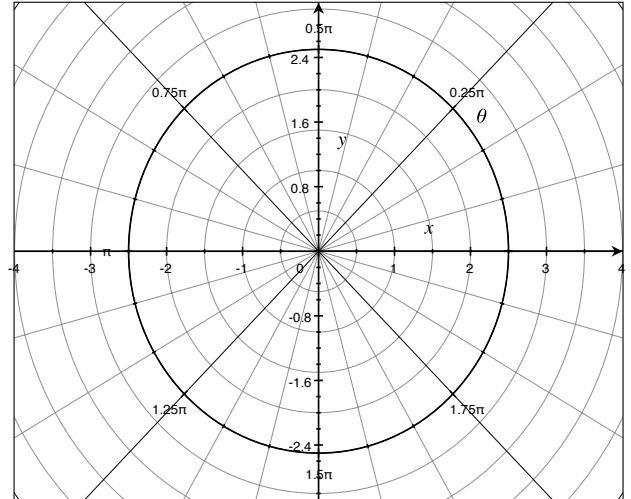
(a) Describe the cross sections created when sliced parallel to the  $xy$  plane. How about for those parallel to the  $yz$  plane.

(b) Find the equation of the tangent plane to the surface at the point  $(1, 1, 1)$ .

7. Let  $P(2, -\frac{2\pi}{3})$  and  $Q(-3, \frac{5\pi}{6})$  be polar coordinates.

(a) [2] Plot  $P$  and  $Q$ .

(b) [2] Find the Cartesian coordinates of the point  $P$ .



(c) [2] Find two other pairs of polar coordinates for the point  $P$ .

8. Consider the function  $h(x, y) = x^3 - 12xy + 8y^3$ .

(a) Find all critical points of  $h$ .

(b) Classify each critical point as a local minimum, a local maximum, or a saddle point.

9. Consider the double integral

$$\int_0^1 \int_x^1 e^{\frac{x}{y}} dy dx$$

(Note: This problem really should be harder. Practice the following questions for the suggested problems from §15.3)

(a) Sketch the region in the  $xy$ -plane where the integral is taken over.

(b) Switch the order of integration.

(c) Compute the double integral.