Exam 1

Tmath 126

Practice

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   (a) If there exists some number M such that $a_n \leq M$ for all n, then $\{a_n\}$ converges.

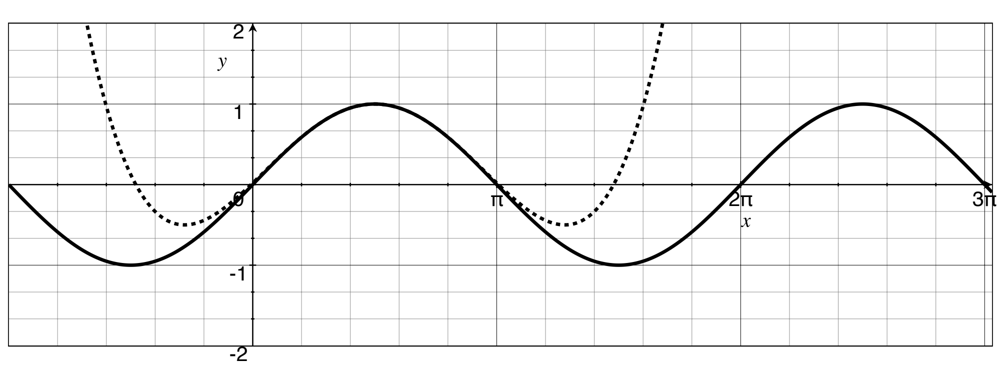
   (b) Every point on the complex plane has a set of polar coordinates which are unique.

   (c) If $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent, then $\displaystyle\lim_{n\to\infty} a_n = 0$.

   (d) The Taylor series is an example of a power series.

   (e) Given a function f, the associated taylor series T has the property that $f(x) = T(x)$ for all x.

   (f) The dotted function below is the 4^{th} Taylor polynomial of $\sin(x)$ centered at 0.



Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Write the following sum in expanded form and simplify: $\displaystyle\sum_{n=1}^{4} \frac{\sqrt{2n+1}}{n!}$

3. Write the following sum using the sigma notation: $1 - \frac{2}{3} + \frac{3}{9} - \frac{4}{27} + \frac{5}{81}$

4. Compute the following if possible.

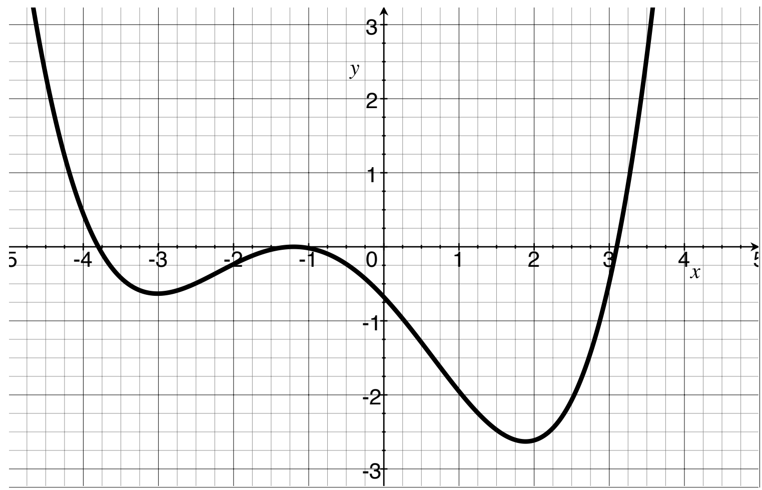
$$\lim_{n\to\infty} \frac{10^{n+1}}{9^n}$$

$$\sum_{n=1}^{\infty} \frac{(2e)^n}{6^{n-1}}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{2}{n}$$

5. Let $p(x) = 0.04 * x^4 + 0.124 * x^3 - 0.3464 * x^2 - 1.09056 * x - 0.678528$ whose graph is shown to the right.



(a) If you wanted to use Newton's method to find the positive root of the function p, what would your first guess be (x_1)?

(b) Using x_1 you choose in part (a), use Newton's method to find x_2.

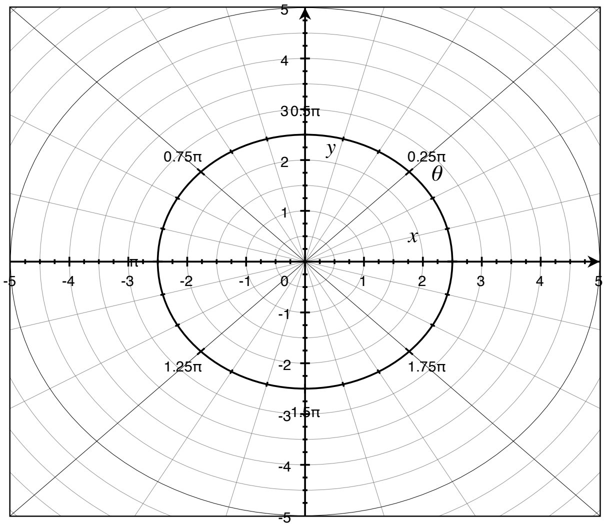
(c) Identify the basins of convergence with Newton's method on the graph.

(d) Find the second order Taylor polynomial $T_2(x)$ based at $b = 1$.

(e) Bound the error $|p(x) - T_2(x)|$ on the interval [0.5,1.5].

6. Consider the point $Q = e^{\frac{i13\pi}{3}}$

(a) Simplify Q so that the angle is between 0 and 2π and then plot Q on the axis provided.



(b) Write Q with rectangular coordinants.

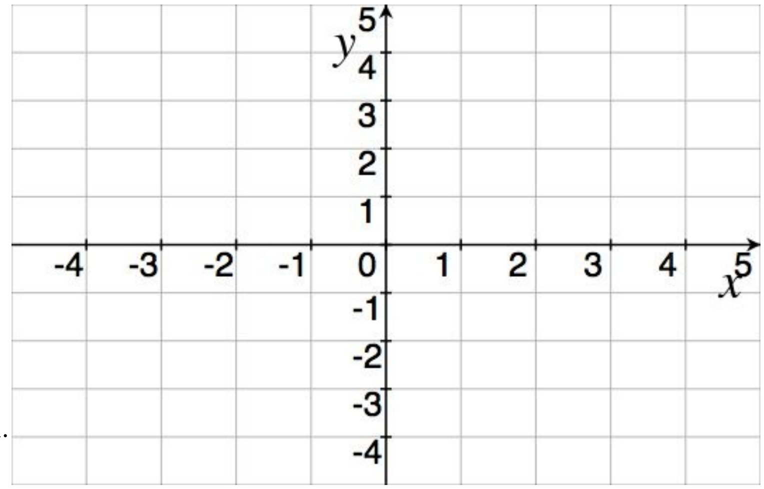
(c) Find $Q^6 - 1$.

(d) Find two solutions of $x^6 = 1$.

(e) ExtraCredit: The Fundamental Theorem of Algebra implies there are six solutions of $x^6 = 1$. Find the other 4.

7. Find the Taylor series expansion for $\dfrac{x}{4+x}$ centered at 0, and find out where it converges.

8. Consider the recursively defined sequence
$\{a_1, \frac{1}{2}(a_1^2-1), \frac{1}{2}\left((\frac{1}{2}(a_1^2-1))^2-1\right), \frac{1}{2}\left(\frac{1}{2}\left((\frac{1}{2}(a_1^2-1))^2-1\right)^2-1\right), \ldots\}$.



(a) Identify the recursive function $R(x)$ so that $a_{n+1} = R(a_n)$.

(b) If $a_1 = -2$, does the resulting sequence converge? If so, identify what it converges to (either find the number or identify it on a graph). Be sure to show your work.

(c) If $a_1 = -3$, does the resulting sequence converge? If so, identify what it converges to (either find the number or identify it on a graph). Be sure to show your work.

(d) How many different limits can the sequence of a_n converge to? Justify yourself.

9. Use geometric series to show $0.99999\ldots = 1$.