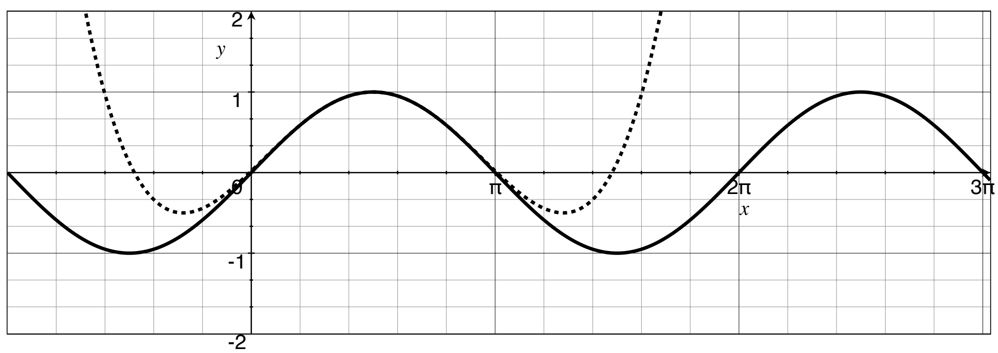
Final **Tmath 126** Summer 2011

1. [12] TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) A series is an "infinite sum" and never converges to a finite number.

   (b) The dotted function below is the 4^{th} Taylor polynomial of $\sin(x)$ centered at 0.

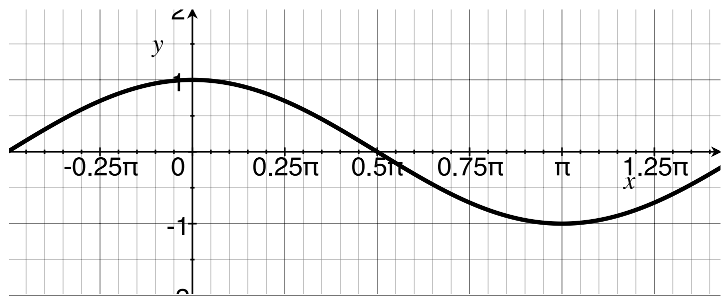


   (c) $(\overrightarrow{a} \times \overrightarrow{b}) \cdot \overrightarrow{a} = 0$.

   (d) $\int_{-1}^{x} \int_{0}^{6} x^2 \sin(x-y)\, dx\, dy = \int_{0}^{6} \int_{-1}^{x} x^2 \sin(x-y)\, dy\, dx$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Recall: $\cos x = \displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$



   (a) [1] Choose an initial value so that Newton's method will not find a root of $\cos(x)$.

   (b) [2] Choose an initial value so that Newton's method does converge to a root and identify the first three approximations on the graph to the right.

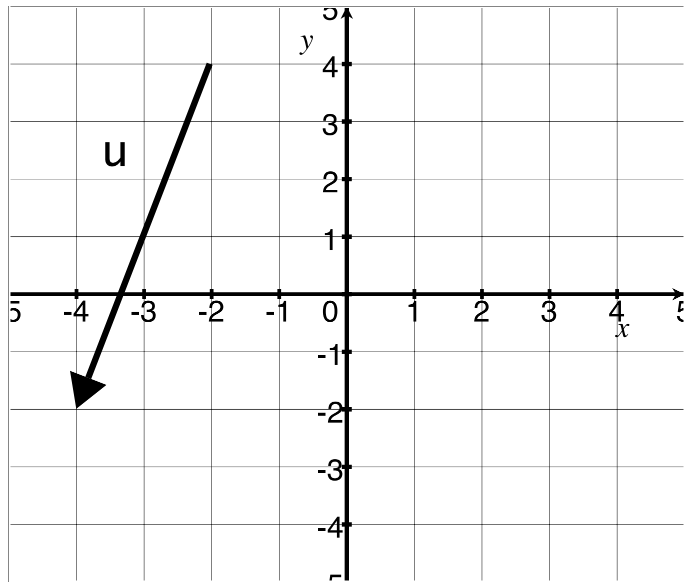
   (c) [2] Find whether the *sequence* $a_n = (-1)^n \dfrac{\left(\frac{\pi}{4}\right)^{2n}}{(2n)!}$ converges or diverges. If it converges, find the limit. *Justify* your answer.

   (d) [5] Find the second order Taylor approximation of $5\cos x$ centered at π.

   (e) [1] Approximate $5\cos 3$ using your above Taylor approximation.

   (f) [4] Use Taylor's inequality to find an upper bound for the error in the approximation above.

3. The vector $\overrightarrow{u} \in \mathbb{R}^2$ and shown below, answer the following:



(a) [1] What are the components of $\overrightarrow{u}$?

(b) [1] Draw the vector $\overrightarrow{v} = \langle 1, -3 \rangle$.

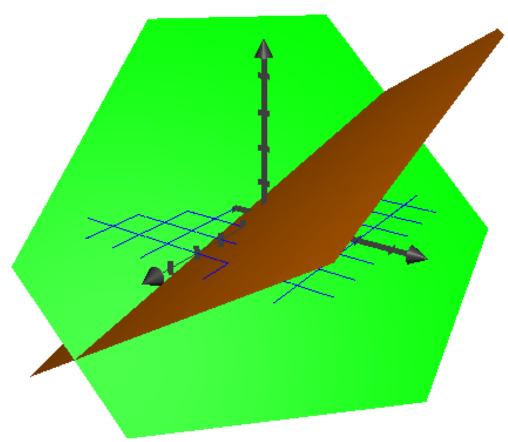
(c) [1] Find $||\overrightarrow{u}||$.

(d) [2] Draw the vector $\overrightarrow{u} + 3\,\overrightarrow{i}$.

(e) [2] Find the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$.

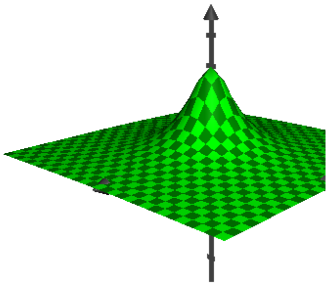
4. Consider the planes P and Q defined by $x+y+z=-1$ and $x-2y+2z=1$ respectively and drawn below.

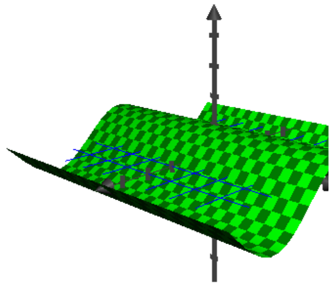
[4] Notice that the two planes intersect and form a line. Find an equation of the line of intersection.

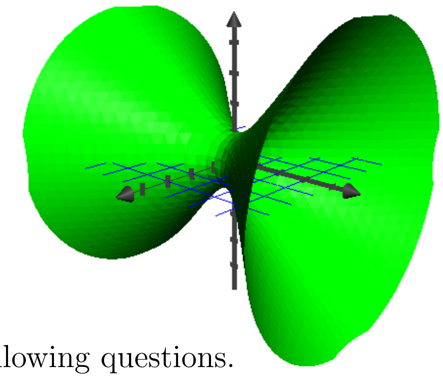


5. [3] Match the following equations to their respective graphs:

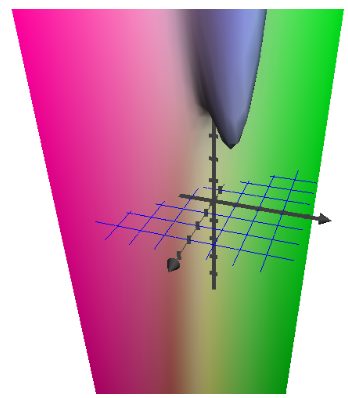
A. $z = \cos x$ B. $x^2 - y^2 + z^2 = 1$ C. $z = \dfrac{3}{1 + x^2 + y^2}$





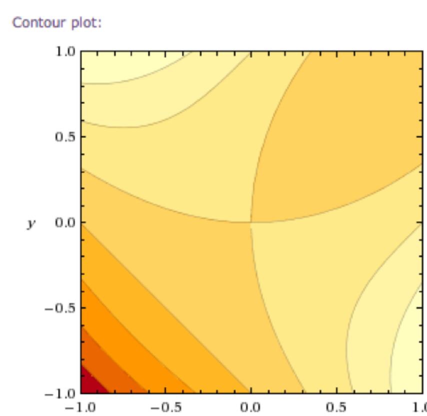


6. Consider the function $f(x, y) = 4 + x^3 + y^3 - 3xy$ for the following questions. The graph and contour lines are provided below.



(a) [3] Find the gradient of f.

(b) [3] Find the equation of the plane tangent to the surface when $x = 2$ and $y = 1$.



(c) [4] Find all critical points and classify them as local minimums, maximums, or saddle points.

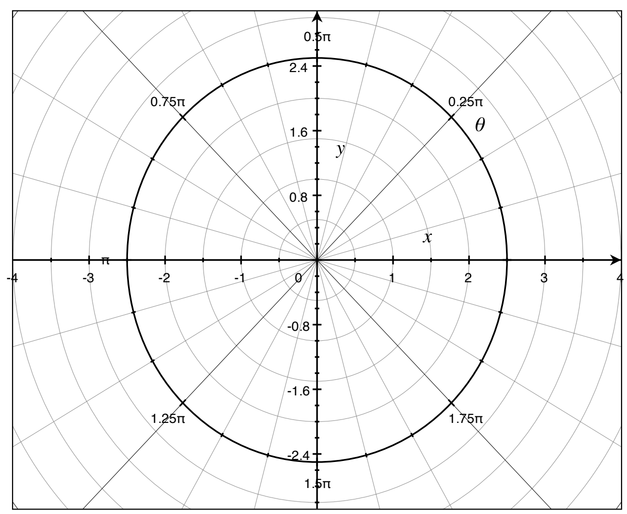
7. [4] Find $\int_0^1 \int_0^2 xye^{x^2 y}\, dy\, dx$.

8. [3] Consider the volume under the surface of the curve $z = xy$ and above the triangle with vertices $(1, 1)$, $(4, 1)$, and $(1, 2)$. Set up the definite integral to find this volume but *do not* integrate.

9. Let $P(2, -\frac{2\pi}{3})$ and $Q(-3, \frac{5\pi}{6})$ be polar coordinates.

(a) [2] Plot P and Q.

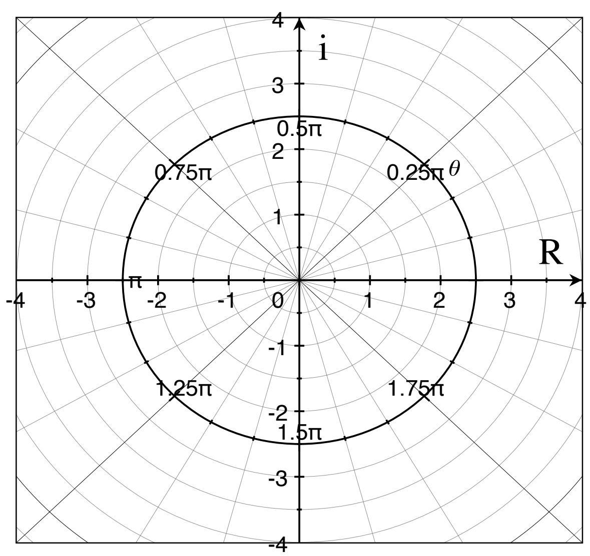
(b) [2] Find the Cartesian coordinates of the point P.



(c) [2] Find two other pairs of polar coordinates for the point P.

10. Let P and Q be complex numbers where $R = e^{\frac{i\pi}{6}}$ and $S = -2 + (2\sqrt{3})i$

(a) [1] Convert R to rectangular coordinates (that is, of the form $a + bi$).



(b) [1] Convert S to polar form (that is, of the form $re^{i\theta}$).

(c) [1] Plot R and S on the complex plane provided to the right.

(d) [1] Multiply $R \cdot S$. (Note: this question didn't make sense in question 8!)

(e) [2] Multiply P^{20} and then plot this point on the axes provided on the right.