Key

Exam 2 — **Tmath 126** — Summer 2012

/45

1. [12] **TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.**

   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) **(Practice Exam #2) If $P = (1, 3, 2)$ and $R = (3, -1, 6)$ are in $\mathbb{R}^3$, then the vector $\overrightarrow{PR}$ has components $\langle 2, -4, 4\rangle$.**

   start (.5) correct vector (1) sense (1)

   True (.5) $\overrightarrow{PR} = \langle 3-1, -1-3, 6-2\rangle = \langle 2, -4, 4\rangle$

   the diff between the x, y, z-coords are 2, -4, 4 respectively

   (b) **(Summer '11 Exam1 #2f) If $\vec{u} = \langle -2, -6\rangle$ and $\vec{v} = \langle 1, -3\rangle$ in $\mathbb{R}^2$ then the projection of $\vec{v}$ onto $\vec{u}$ is $\langle \frac{-4}{5}, \frac{12}{5}\rangle$.**

   start (.5) knows proj (1) sense (1)

   False (.5) $\text{proj}_{\vec{u}}\vec{v} = \underbrace{\|\vec{v}\|\cos\theta}_{\text{magnitude}} \underbrace{\frac{\vec{u}}{\|\vec{u}\|}}_{\text{direction}}$ Since $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}$

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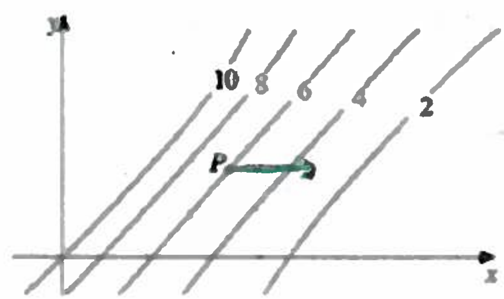
   $= \|\vec{v}\| \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}\frac{\vec{u}}{\|\vec{u}\|} = \frac{\langle -2,-6\rangle\cdot\langle 1,-3\rangle}{(\sqrt{2^2+6^2})^2}\langle -2,-6\rangle = \frac{16}{40}\langle -2,-6\rangle = \frac{2}{5}\langle -2,-6\rangle$

   (c) **(Quiz 3 #1) The volume of a parallelepiped with edges $\overline{PQ}$, $\overline{PR}$ and $\overline{PS}$ can be found by computing $(\overrightarrow{PQ}\cdot\overrightarrow{PR})\times\overrightarrow{PS}$.**

   start (.5) knows $\cdot$ & $\times$ (1) sense (1)

   False (.5) $\overrightarrow{PQ}\cdot\overrightarrow{PR}$ returns a scalar but a cross product needs to be between two vectors. Thus $(\overrightarrow{PQ}\cdot\overrightarrow{PR})\times\overrightarrow{PS}$ doesn't even make sense.

   (d) **(§14.1 #74) Level curves are shown for the function f. From this we know $f_x > 0$.**



   start (.5) knows f_x (1) sense (1)

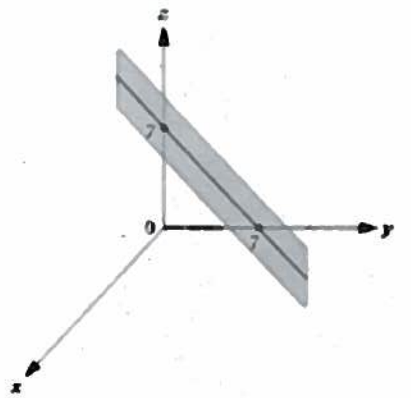
   False (.5). In the graph of contour lines shown as we travel in $\langle 1, 0\rangle$ direction, $\Delta z < 0$

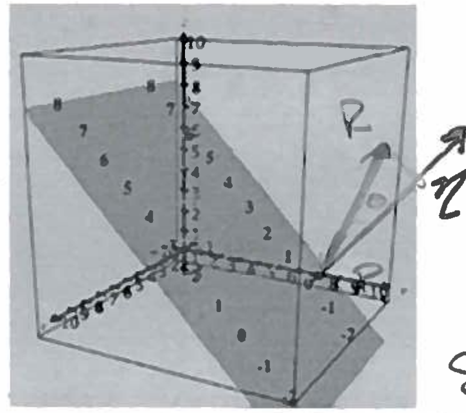
   $\Rightarrow f_x < 0$

   $\frac{\Delta z}{\Delta x}$ $\frac{\text{down}}{1}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. **Consider the surface sitting in three dimensions shown from two views (one with contour lines) below :**





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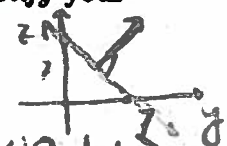
(a) **[1] (3D & vector wks #2) Identify a point (coordinate) that is on the surface.**

(0,7,0) or (0,0,7)

(b) **[2] (PracticeExam #3) Does the surface define z as a function of x and/or y? Why or why not?** Know function (+.5)

(+.5) Yes b/c each (x,y) pair has only one z coord assoc with it
(+1) ie it passes the vertical line test

(c) **[3] (§12.4 #32) Identify a unit vector that is normal/orthogonal/perpendicular to the surface shown. (You need not use calculus here, but be sure to *justify* your answer.)**



justify (+1.5) Since we have a cylinder we can just picture the graphs intersection with the yz plane.
We have an isosceles $\Delta \Rightarrow$ the normal vector is $\langle 0,1,1\rangle$ (+.5)

(+.5) Thus a unit vector is $\frac{1}{\sqrt{2}}\langle 0,1,1\rangle$

(d) **[2] (WebHW7 #2) Write an equation(s) for the surface.**

$\langle 0,1,1\rangle \cdot (\langle x,y,z\rangle - (0,7,0)) = 0$ works!

or $y-7+z=0$ or $z=7-y$

eq of line (+.5)
normal (+.5)
point (+.5)
plug in (+.5)

(e) **[4] (WebHW9 #9) Find the distance from the point $(-1,9,5)$ to the surface shown.**

(+1.5) Where the distance corresponds to $\vec{PR}$ projected onto $\vec{\eta}$

$\vec{PR} = \langle -1-0, 9-7, 5-0\rangle = \langle -1,2,5\rangle$

$\text{Proj}_{\vec{\eta}}\vec{PR} = \underbrace{\|\vec{PR}\|\cos\theta}_{\text{magnitude}} \underbrace{\frac{\vec{\eta}}{\|\vec{\eta}\|}}_{\text{direction}}$

(+.5) ie the magnitude ...

Since $\vec{PR}\cdot\vec{\eta} = \|\vec{PR}\|\|\vec{\eta}\|\cos\theta$

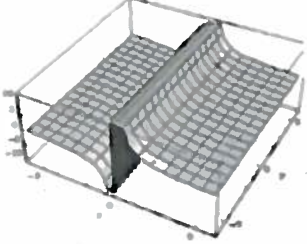
magnitude $= \cancel{\|\vec{PR}\|}\dfrac{\vec{PR}\cdot\vec{\eta}}{\cancel{\|\vec{PR}\|}\,\|\vec{\eta}\|}$ (+.5)

$= \dfrac{\langle -1,2,5\rangle\cdot\langle 0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle}{1}$ (+1)

notation/sense (+.5)

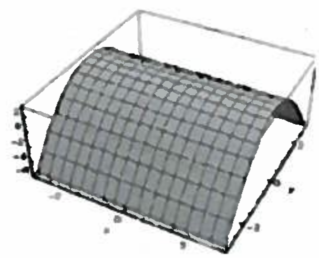
$= \frac{2}{\sqrt{2}} + \frac{5}{\sqrt{2}} = \frac{7}{\sqrt{2}} \approx$

i) A



ii) C

iii) B



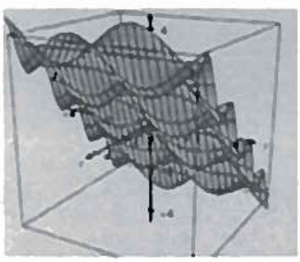
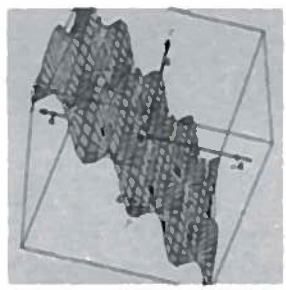
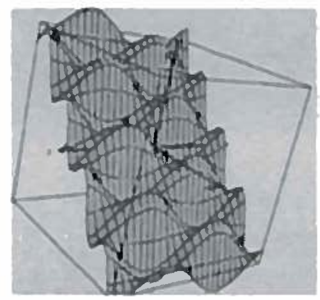
3. [4] (WebHW9 #10,11,14) Consider the three graphs above. Match the following equations to their respective graphs:

A (.5) consider each (+.5)

A. $yz = 6$ (+.1) B. $z = 3 - y^2$ iii (+1) C. $y = 2x^2 + z^2$ (+1)

cylinder (does not depend on x) $z = 6/y$ i — cylinder (does not depend on x) cross sections look like upside down parabolas — notice (0,0,0) satisfies this ii

4. (Summer '11 Exam2 #5) Three views of the function $f(x, y) = x + \cos(3x)\sin(y)$ are shown below and may be used for the following questions. The point $(0, \frac{\pi}{2}, f(0, \frac{\pi}{2}))$ is identified on the graph.

(a) [3] Find the gradient of f.

$\nabla f = \underbrace{\langle f_x, f_y \rangle}_{(+1)} = \langle \underbrace{1 - 3\sin(y)\sin 3x}_{(+1)}, \underbrace{\cos 3x \cos y}_{(+1)} \rangle$

(b) [3] Find the directional derivative of f at the point $(0, \frac{\pi}{2})$ in the direction of $\vec{w} := \frac{1}{\sqrt{5}}(\vec{i} - 2\vec{j})$. Notice $\|\vec{w}\| = \sqrt{\frac{1}{\sqrt{5}}^2 + (\frac{2}{\sqrt{5}})^2} = 1$ so $\vec{w}$ is a unit vector.

$D_{\vec{w}}(f)\big|_{(0,\frac{\pi}{2})} = \underbrace{\vec{w} \cdot \nabla f\big|_{(0,\frac{\pi}{2})}}_{(+1)} = \langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \rangle \cdot \underbrace{\langle 1 - 3\cdot 1\cdot 0, 1\cdot 0 \rangle}_{(+.5)}$

$= \langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \rangle \cdot \langle 1, 0 \rangle = \frac{1}{\sqrt{5}}$ } (+1)

(c) [3] (§14.4 #16) Find the linearization of f at the point $(0, \frac{\pi}{2})$.

(+.5) { Looking for $z - z_1 = m_x(x - x_1) + m_y(y - y_1)$

(+1.5) { $m_x = f_x(0, \frac{\pi}{2}) = 1 - 3\cdot 1\cdot 0 = 1$
$m_y = f_y(0, \frac{\pi}{2}) = 1 \cdot 0 = 0$

(+.5) { thru $(0, \frac{\pi}{2}, 0 + \cos(0)\sin(\frac{\pi}{2})) = (0, \frac{\pi}{2}, 1)$

So $z - 1 = 1(x - 0) + 0(y - \frac{\pi}{2})$

plug in (+.5)

$\Rightarrow$

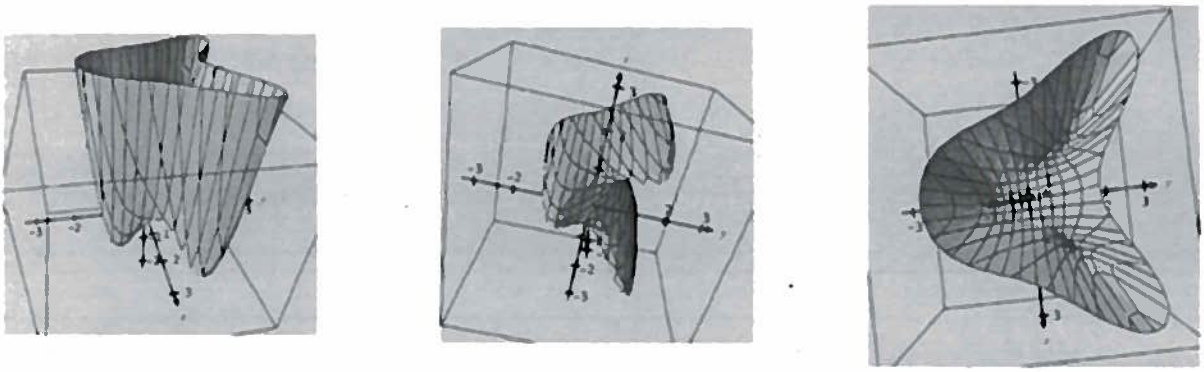
$z - 1 = x$

5. (§14.7 #25) Consider the function $g(x, y) = x^4 + y^4 - 4x^2y + 2y$. Three views of the function g are shown below.

(a) [3] ***Clearly outline*** **steps to identify all critical points and then classify them as local minimums, local maximums, or saddle points.** The outline should make clear what critical points are

add →

(b) **[5] Perform the above steps to find all the critical points and classify them.**



(a) Step 1: find the critical points (CP)

a) find g_x and g_y

b) find the zeros of g_x & g_y as well as domain restrictions that g does not share.

note: this may use Newton's method

Step 2 (+1)

CP (+1)

sense (+1)

Step 2: examine each CP to determine if max, min or saddles by using one of following:

a) 2nd derivative test

b) examine given graph

(b) Step 1

(+1)

a) $g_x(x,y) = 4x^3 - 8xy$

$g_y(x,y) = 4y^3 - 4x^2 + 2$

b) ∄ domain restrictions so we look for zeros

algebra (+1)

$$\begin{Bmatrix} 4x^3 - 8xy = 0 \\ 4y^3 - 4x^2 + 2 = 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} 4x(x^2 - 2y) = 0 \\ 4y^3 - 4x^2 + 2 = 0 \end{Bmatrix}$$

1st eq $\Rightarrow x = 0$ or $x^2 = 2y$

2nd eq $\Rightarrow y = \sqrt[3]{\frac{-1}{2}}$ $\quad 4y^3 - 4(2y) + 2 = 0$

(+.5)

To find the roots of $4y^3 - 8y + 2 = 0$ we can use Newton's method

i) choose a guess a_1

ii) $a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)} = a_n - \frac{4a_n^3 - 8a_n + 2}{12a_n^2 - 8}$

(b/c we are looking for intercept of tang line

$b - b_1 = m(a - a_1)$ $\quad m = f'(a_1)$

and intercept $\Rightarrow b = 0$

$0 - b_1 = f'(a_1)(a - a_1) \Rightarrow \frac{-b_1}{f'(a_1)} = a - a_1$

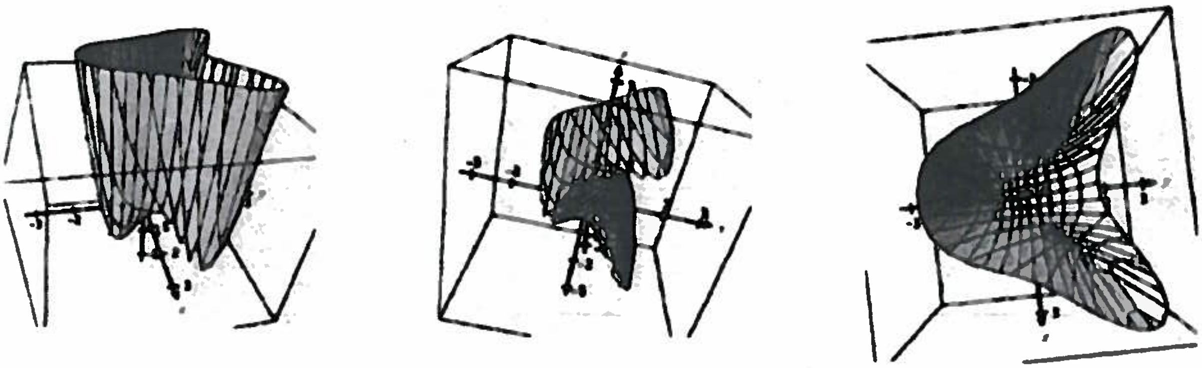
$\Rightarrow a = a_1 - b_1/f'(a_1) = a_1 - \frac{f(a_1)}{f'(a_1)}$)

con't

5. (§14.7 #25) Consider the function $g(x, y) = x^4 + y^4 - 4x^2y + 2y$. Three views of the function g are shown below.

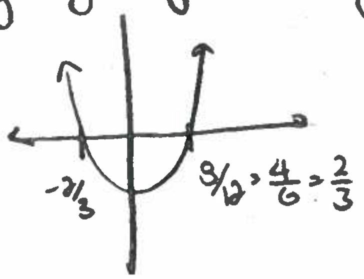
   (a) [3] *Clearly outline* steps to identify all critical points and then classify them as local minimums, local maximums, or saddle points.

   (b) [5] Perform the above steps to find all the critical points and classify them.

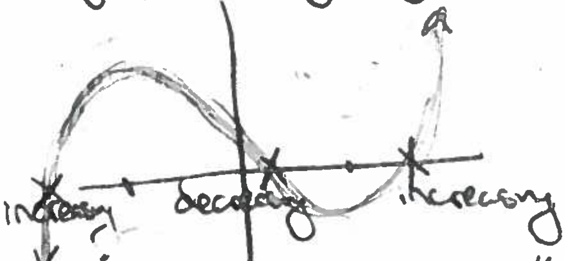


I dont have a calculator but

$\frac{d}{dy}(4y^3 - 8y + 2) = 12y^2 - 8$



$\Rightarrow$ graph of $4y^3 - 8y + 2$ looks like



try $a_1 = -1 \rightsquigarrow \{-1, -1 - \frac{4(-1)+8+2}{12(-1)^2-8},$

$a_1 = 0 \rightsquigarrow \{0, 0 - \frac{0-0+2}{0-8}, \ldots$

$a_1 = 1 \rightsquigarrow \{1, 1 - \frac{4(1)-8(1)+2}{12(1)-8} \ldots$

$\{-1, -5/2, -1.896, -1.609, -1.53 \ldots\}$

$\{0, 1/4, .2586, .2587, \ldots\}$

$\{1, 3/2, 1.316, 1.27, 1.267, \ldots\}$

So y could be -1.55, $.26$, or 1.27

1.5

which means the CP are: (b/c $x^2 = 2y \Rightarrow x = \pm\sqrt{2y}$)

min $(0, \sqrt[3]{1/8})$, saddle $(\sqrt{2 \cdot .26}, .26)$, saddle $(-\sqrt{2 \cdot .26}, .26)$,

min $(\sqrt{2 \cdot 1.27}, 1.27)$, min $(-\sqrt{2 \cdot 1.27}, 1.27)$

inspecting the graph then we can determine which CP is max/min/saddle

+1