→ d'Souza

Exam 1 # Tmath 126 Summer 2012

[11] 1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

~~(a) (quiz1 #1) If $p \geq 1$, then the sequence $a_n = \left(\frac{1}{n}\right)^p$ converges.~~

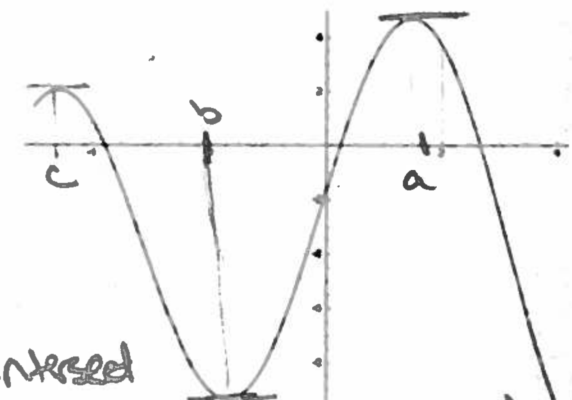
start (+.5)
reasoning/sense (+1)
exp limits (+1)

True (+.5) since $\lim_{n\to\infty} \left(\frac{1}{n}\right)^p = \left(\lim_{n\to\infty} \frac{1}{n}\right)^p$

since "$\lim_{Big\to\infty} \frac{1}{Big} = 0$" the above equals

$0^p = 0$

(b) We can choose any starting point x_0 for Newton's method to find a root of $7\sin(x) - \sqrt{x^2+3}$.



start (+.5)
looking for counterex (+1)
reasoning/sense (+1)

false (+.5), points a, b, & c have horizontal tangents & so never intersect the x-axis ⇒ there is no next step in Newton's method!

(c) (pg778 #1) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

start (+.5)
looking for counterex (+1)
reasoning/sense (+1)

false (+.5) Consider $\sum_{n=1}^{\infty} \frac{1}{n}$ ie the harmonic series

Notice $\lim_{n\to\infty} a_n = 0$ but $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges to ∞

(d) (Lecture 2/27) The first degree Taylor polynomial of a function f centered at 2 is the same as the line tangent to f when x=2.

start (+.5)
reasoning/sense (+1)
definitions (+1)

true (+.5) by construction, the first Taylor polynomial is so that $T_1(2) = f(2)$ and $T_1'(2) = f'(2)$ which gives rise to the line tangent to f at $x=2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [5] (§11.10 #20) Write the following sum using the sigma notation:

Fix $\longrightarrow$

$$-4+\frac{1}{8}(x-16)+\frac{-1}{2!\cdot 256}(x-16)^2+\frac{1}{3!\cdot 8192}(x-16)^3+\frac{-1}{4!\cdot 262144}(x-16)^4$$

$$-2^2+\frac{1}{2^3}(x-16)-\frac{(x-16)^2}{2!\cdot 2^8}+\frac{(x-16)^3}{3!\cdot 2^{13}}-\frac{(x-16)^4}{4!\cdot 2^{18}}$$

$$-2^2+\sum_{n=1}^{4}\frac{(-1)^{n+1}(x-16)^n}{n!\; 2^{5n-2}}=\sum_{n=0}^{4}\frac{(-1)^{n+1}(x-16)^n}{2^{5n-2}}$$

sigma +.5
start +.5
stop +.5
$(-1)^n$ +.5
$(x-16)^n$ +.5
$n!$ +.5
power of 2 +.5
constant +.5

3. Compute the following if possible.

(a) [4] (WebHW1 #8) $\lim_{n\to\infty} a_n$ where $a_n=\tan\left(\frac{2n\pi}{1-12n}\right)$

$$\lim_{n\to\infty} a_n=\lim_{n\to\infty}\tan\left(\frac{2n\pi}{1-12n}\right)$$ +1 prop of limits

$$=\tan\left(\lim_{n\to\infty}\frac{2n\pi}{1-12n}\right)\underset{L'H}{=}\tan\left(\lim_{n\to\infty}\frac{2\pi}{-12}\right)$$ +1

$$=\tan\left(\frac{-\pi}{6}\right)=\frac{-1/2}{\sqrt{3}/2}=-1/\sqrt{3}$$ +1

working with sequence +1

(b) [4] (§11.2 #24) The series $\sum_{n=0}^{\infty} a_n$ where $a_n=\left(\frac{1}{\sqrt{2}}\right)^n$

$$\sum_{n=1}^{\infty} a_n=1+\frac{1}{\sqrt{2}}+\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^3+\left(\frac{1}{\sqrt{2}}\right)^4+\cdots$$ +1

geometric series

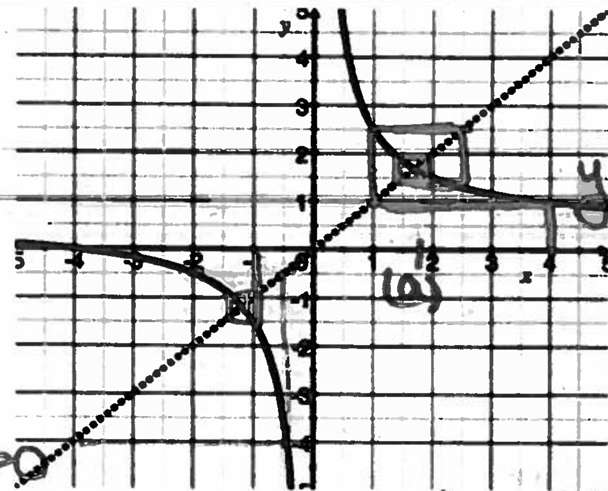
$a=1 \quad r=\frac{1}{\sqrt{2}}$ +1

converges to $\frac{a}{1-r}=\frac{1}{1-\frac{1}{\sqrt{2}}}=\frac{\sqrt{2}}{\sqrt{2}-1}$ +1

working with sum +1

label graph →

4. (WebHW2 #2) Consider the recursively defined sequence $\{a_n\}_{n=1}^{\infty}$ where $a_{n+1} = \dfrac{2}{a_n} + \dfrac{1}{2}$.



$y = \dfrac{2}{x} + \dfrac{1}{2}$

(a) [3] If $a_1 = 4$, does the resulting sequence converge? If so, identify what it converges to (either find the number or identify it on a graph). Be sure to show your work.

yes, converges to (a)

cobweb algorithm (+2) interpret (+1)

or

(+1) $\left\{ L = \dfrac{2}{L} + \dfrac{1}{2} \Rightarrow L^2 - .5L - 2 = 0 \right.$

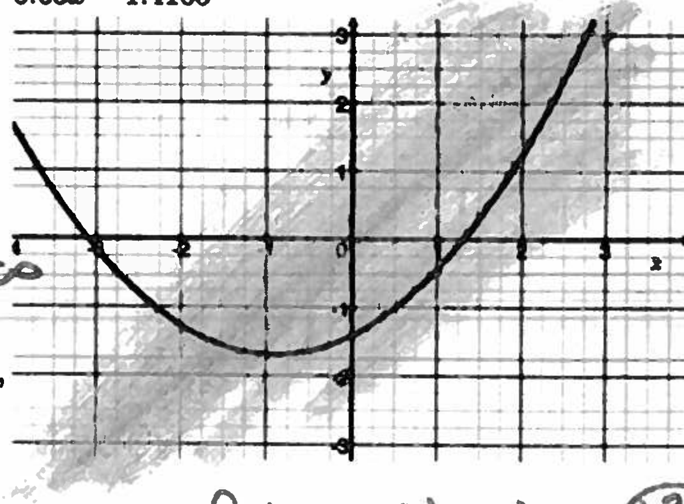
(+1.5) $\left\{ \Rightarrow L = \dfrac{.5 \pm \sqrt{.25 + 4(1)2}}{2(1)} = \dfrac{.5 \pm \sqrt{8.25}}{2} \right.$ positive one by graph (+.5)

(b) [2] If possible, choose an a_1 so that the resulting sequence $\{a_n\}_{n=1}^{\infty}$ converges to a negative value. If not possible, explain why.

-1 should work. (+1)

see cobwebbing (+1)

5. (PracticeExam #5) Let $p(x) = 0.35x^2 + 0.63x - 1.4105$ whose graph is shown to the right.



(a) [1] If you wanted to use Newton's method to find the positive root of the function p, what would your first guess be (x_1)?

anything between -.8 to ∞

(b) [4] Using x_1 you choose in part (a), use Newton's method to find x_2.

(+1) $p'(x) = .7x + .63x$

(+.5) $p'(x_1)$

(+.5) $p(x_1)$

(+2) $\left\{ x_2 = x_1 - \dfrac{p(x_1)}{p'(x_1)} = \right.$

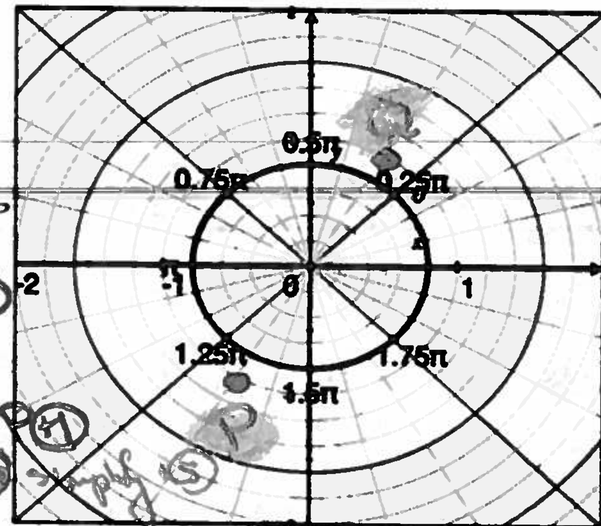
find eq of tang line (+3)

or find the x intercept (+1)

6. Consider the points $Q = e^{\frac{i13\pi}{3}}$ and $P = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$.

$$e^{\frac{i13\pi}{3}} = e^{\frac{12\pi}{3}i} e^{\frac{\pi i}{3}} = e^{\pi i/3}$$

(a) [2] Plot P and Q. (+1) (+1)



(b) [2] (~~PracticeExam #6~~) Convert Q into rectangular coordinates.

Sohcahtoa (+1) $r=1$ $\sin\frac{\pi}{3} = \frac{y}{1}$, $\cos\frac{\pi}{3} = \frac{x}{1}$ $\Big\}$ $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ (+.5) (+.5)

(c) [2] (WebHW3 #4) Find $P \cdot Q$. Write your answer in either polar or rectangular coordinates, but *simplify*.

$P \cdot Q = e^{4\pi i/3} e^{\pi i/3} = e^{5\pi i/3}$ $\Big\}$ convert P (+1) alg (+.5) simplify (+.5)

or $\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) =$

$\Big\}$ Foil (+1) simplify (+1)

~~(d) [2] (ApxH #34) Find Q^5 and simplify.~~

7. Consider the function $f(x) = \ln(5 - x)$

(a) [5] (§11.9 #15) Find a power series representation for f and determine the radius of convergence. Any power series will suffice but supply work so I can see which one you are working with. (+1)

+3 Recall $\ln(1+u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \dots$ using the Taylor series $(b=0)$

$\ln(5-x) = \ln 5\left(1 + \frac{-x}{5}\right) = \ln 5 + \ln\left(1 + \frac{-x}{5}\right)$ (+1.0)

(+1.0) $= \ln 5 + \left(\frac{-x}{5} - \left(\frac{x}{5}\right)^2\frac{1}{2} - \left(\frac{x}{5}\right)^3\frac{1}{3} - \left(\frac{x}{5}\right)^4\frac{1}{4} \dots\right)$ (+.5)

+2 Since ~~$\ln(1+u)$ had a radius of convergence of 1~~ $(-1 < u < 1)$
we have $-1 < \frac{-x}{5} < 1 \Rightarrow 5 > x > -5$ so $R = 5$ (+1) (+.5)

(b) [4] Find a *reasonable* bound for the error of the second Taylor polynomial approximation centered at 0 for $f(3.5)$. Make sure that you show enough work that I know why you choose the M that you did.

Recall error of T_2 is bound by $\frac{M}{3!}(x-b)^3$ $\Big\}$ (+.5)

M is such that $M < |f^{(3)}(x)|$ for $x \in (0, 3.5)$

$M = f^{(3)}(\ldots)$ works so

$(3.5 - 0)^3$

n	$f^{(n)}(x)$
0	$\ln(5-x)$
1	$\frac{-1}{5-x} = -(5-x)^{-1}$
2	$\frac{-1}{(5-x)^2} = -(5-x)^{-2}$
3	$\frac{-2}{\dots}$

(+1)