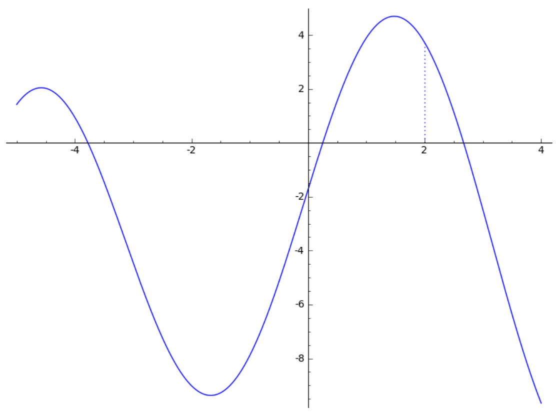
Exam 1 | Tmath 126 | Summer 2012

1. [12] TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   (a) (quiz1 #1) If $p \geq 1$, then the sequence $a_n = \left(\dfrac{1}{n}\right)^p$ converges.

   (b) We can choose any starting point x_0 for Newton's method to find a root of $7\sin(x) - \sqrt{x^2+3}$.



   (c) (pg778 #1) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (d) (Lecture 2/27) The first degree Taylor polynomial of a function f centered at 2 is the same as the line tangent to f when x=2.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [5] (§11.10 #20) Write the following sum using the sigma notation:

$$-4+\frac{1}{8}(x-16)+\frac{-1}{2!\cdot 256}(x-16)^2+\frac{1}{3!\cdot 8192}(x-16)^3+\frac{-1}{4!\cdot 262144}(x-16)^4$$

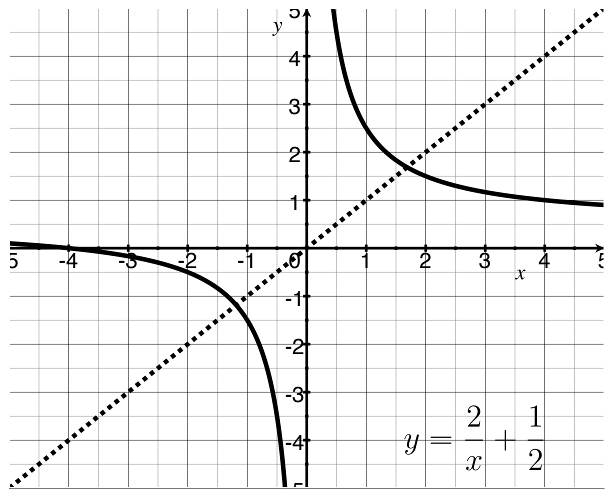
3. Compute the following if possible.

   (a) [4] (WebHW1 #8) $\lim_{n\to\infty} a_n$ where $a_n = \tan\left(\dfrac{2n\pi}{1-12n}\right)$

   (b) [4] (§11.2 #24) The series $\sum_{n=0}^{\infty} a_n$ where $a_n = \left(\dfrac{1}{\sqrt{2}}\right)^n$

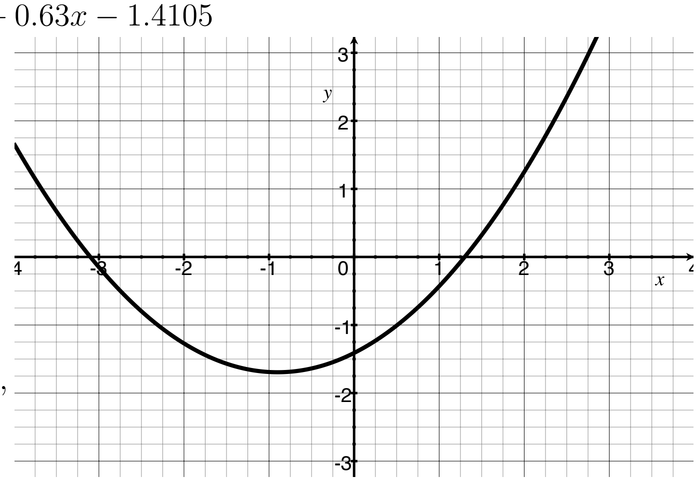
4. (WebHW2 #2) Consider the recursively defined sequence $\{a_n\}_{n=1}^{\infty}$ where $a_{n+1} = \dfrac{2}{a_n} + \dfrac{1}{2}$.

   (a) [3] If $a_1 = 4$, does the resulting sequence converge? If so, identify what it converges to (either find the number or identify it on a graph). Be sure to show your work.



   (b) [2] If possible, choose an a_1 so that the resulting sequence $\{a_n\}_{n=1}^{\infty}$ converges to a negative value. If not possible, explain why.

5. (PracticeExam #5) Let $p(x) = 0.35x^2 + 0.63x - 1.4105$ whose graph is shown to the right.



   (a) [1] If you wanted to use Newton's method to find the positive root of the function p, what would your first guess be (x_1)?

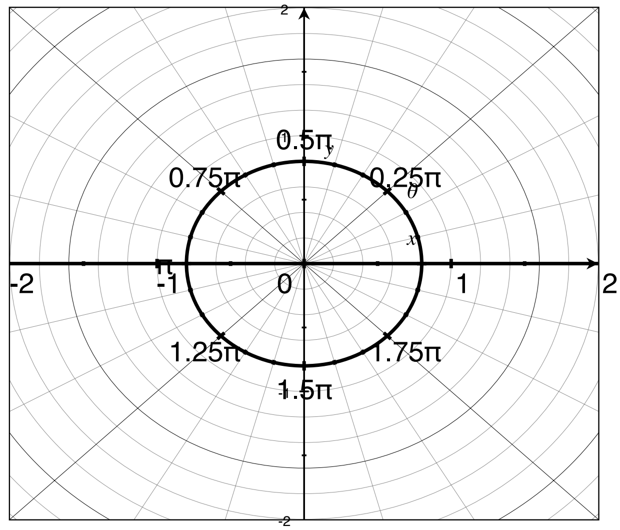
   (b) [4] Using x_1 you choose in part (a), use Newton's method to find x_2.

6. Consider the points $Q = e^{\frac{i13\pi}{3}}$ and $P = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$.

(a) [2] Plot P and Q.

(b) [2] (PracticeExam #6) Convert Q into rectangular coordinates.

(c) [2] (WebHW3 #4) Find $P \cdot Q$. Write your answer in either polar or rectangular coordinates, but *simplify*.



7. Consider the function $f(x) = \ln(5 - x)$

(a) [5] (§11.9 #15) Find a power series representation for f and determine the radius of convergence. Any power series will suffice but supply work so I can see which one you are working with.

(b) [4] Find a *reasonable* bound for the error of the second Taylor polynomial approximation centered at 0 for $f(3.5)$. Make sure that you show enough work that I know why you choose the M that you did.