§14.4 Tangent Lines & Planes

Recall any of the following could be used to describe a line in $\mathbb{R}^2$:

$$y = mx + b \qquad ax + by = c$$

$$y - y_1 = m(x - x_1).$$

Consider an example of my favorite type of differential calculus question:

1. Find the line tangent to the graph of $f(x) = 2x^2$ when x=1.

2. Find the local linearization of f when $x = 1$.

3. Use the linearization of f at $(1, 2)$ to approximate $f(1.1)$.

4. How good is the approximation above? That is, what is the difference between your approximation above, and the actual value $f(1.1)$.

Recall any of the following could be used to describe a plane in $\mathbb{R}^3$:

$$\overrightarrow{n} \cdot (\langle x, y\rangle - \langle x_1, y_1\rangle \qquad ax + by + cz = d$$

$$z - z_1 = m_x(x - x_1) + m_y(y - y_1).$$

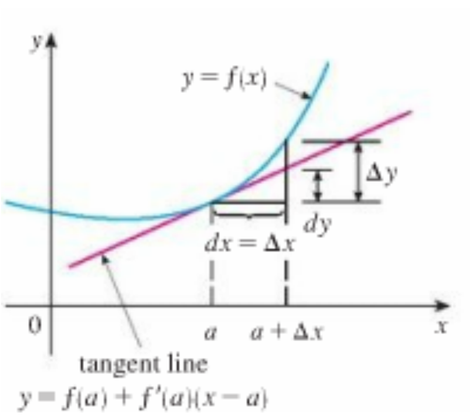
1. Find the plane tangent to the graph of $f(x, y) = 2x^2 + y^2$ when $x = 1$ and $y = 1$.

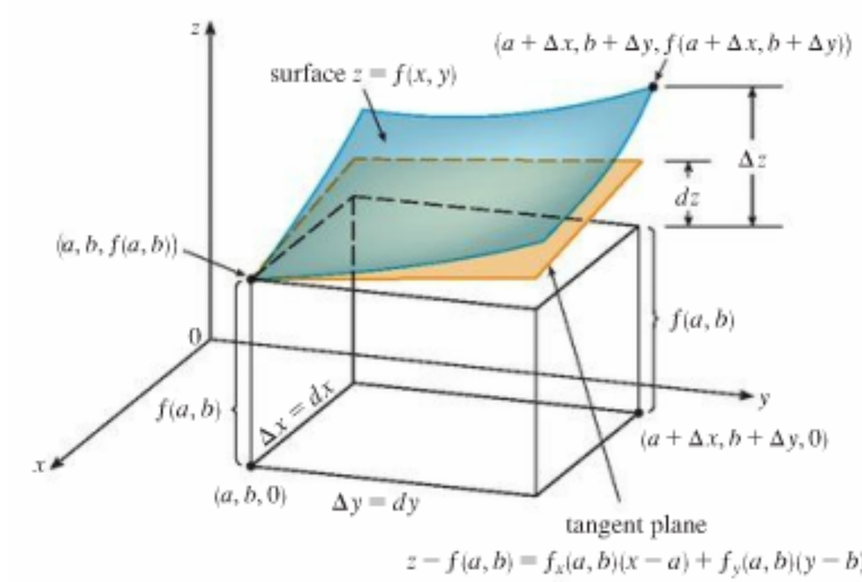
2. Find the local linearization of f when $x = 1$ and $y = 1$.

3. Use the linearization of f at $(1, 1, 3)$ to approximate $f(1.1, 1.1)$.

4. How good is the approximation above? That is, what is the difference between your approximation above, and the actual value $f(1.1, 1.1)$.

Total Differential: df





§14.6 Directional Derivatives

1. Let $f(x, y) = x + e^y$.

   (a) Find ∇f

   (b) Find $D_{\overrightarrow{u}} f(1, 1)$ where $\overrightarrow{u} = \langle 1, -1 \rangle$.

   (c) Find $D_{\overrightarrow{u}} f(1, 1)$ where $\overrightarrow{u} = \overrightarrow{i} + 2\overrightarrow{j}$.