TMATH 126: Quiz 3

Key

You may use any work of yours that you made from last week. This includes, practice problems from the book and worked out WebAssign problems. This *does not* include photocopies of notes from the book or tutorials shown on WebAssign. You may also use a calculator, but you are not allowed to use any device that can access the internet.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

+.5 started
+.5 truth value
+.5 appropriate stmt/ex
+.5 got it

T F To prove that a statement is true, I need only show an example.

Statements can sometimes be true, but not always true.
example: "Any number times one is one. ex $1\cdot1=1$"
but the statement is not always true b/c $5\cdot1=5$ ≠.

T F Let f and g be functions of x and y. The ordered pair (a, b) is a maximum or minimum if the vector $\nabla f(a, b)$ is parallel to $\nabla g(a, b)$.

Consider the contour lines of f & g. The countour lines will become tangent when the extrema value is reached implying the perpendiculars (to the countour curve in the x-y plane) ie $\nabla f(a,b)$ & $\nabla g(a,b)$ are parallel.

Note: I did assume that ∇f + ∇g existed

T F The system of equations below has two equations and two unknowns, but we cannot solve for x or y.

Yes we can!

$$0 = -6x(x^2 + y^2)e^{y^2-x^2} + 6xe^{y^2-x^2}$$
$$0 = 6y(x^2 + y^2)e^{y^2-x^2} + 6ye^{y^2-x^2}$$

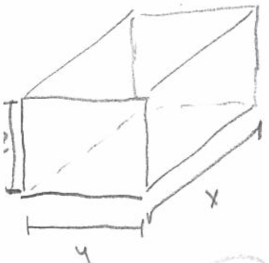
$\Rightarrow \begin{cases} 0 = 6xe^{y^2-x^2}(-x^2-y^2+1) \\ 0 = 6ye^{y^2-x^2}(x^2+y^2+1) \end{cases}$

eq 1 $\Rightarrow$ $6x=0$ or $e^{y^2-x^2}=0$ or $-x^2-y^2+1=0$
$\Rightarrow x=0$ never happens $\Rightarrow (*)\ x^2+y^2=1 \Rightarrow (x,y)$ is on the unit

eq 2 $\Rightarrow$ $6y=0$ or $e^{y^2-x^2}=0$ or $x^2+y^2+1=0$
$\Rightarrow y=0$ never happens never happens

So $y=0$. Then $x=0$ or $x^2+0^2=1 \Rightarrow x=\pm1$.
$(0,0)$ $(-1,0)$ + $(1,0)$ are solutions.

2. The base of an aquarium with given volume of 400 ft^3 is made of slate and the sides are made of glass. The slate costs three times as much (per unit area) as glass (there is no top). We want to minimize the cost of the materials.



(a) [2] Determine the function that you would like to minimize as a function of the dimensions of the aquarium. Be sure to clearly define your variables if you create them.

3 times price

Cost of front + Cost of side + Cost of side + Cost of back + cost of bottom

$= zyp + zxp + zxp + zyp + 3pxy$

$= p(2zy + 2zx + 3xy)$ where p is the price, but we could ignore p & instead minimize

$f(x,y,z) = 2zy + 2zx + 3xy$

variables (+.5)

(b) [2] Determine any constraints that this problem has.

$400 = x \cdot y \cdot z$

(c) [4] Set up the system of equations to find the minimum, but *do not* solve them.

$\begin{cases} \nabla f(x,y,z) = \lambda \nabla g(x,y,z) \\ g(x,y,z) = 400 \end{cases}$ where $g(x,y,z) = x \cdot y \cdot z$

$\Rightarrow \begin{cases} f_x(x,y,z) = \lambda g_x(x,y,z) \\ f_y(x,y,z) = \lambda g_y(x,y,z) \\ f_z(x,y,z) = \lambda g_z(x,y,z) \\ g(x,y,z) = 400 \end{cases} \Rightarrow \begin{cases} 2z + 3y = \lambda yz \\ 2z + 3x = \lambda xz \\ 2y + 2x = \lambda xy \\ xyz = 400 \end{cases}$

3. [6] Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2y$ subject to the constraint $x^2 + 2y^2 = 6$.

$f(x,y) = x^2y$
$g(x,y) = x^2 + 2y^2$

$\begin{cases} \nabla f(x,y) = \lambda \nabla g(x,y) \\ g(x,y) = 6 \end{cases} \Rightarrow \begin{cases} 2xy = \lambda 2x \\ x^2 = \lambda 4y \\ x^2 + 2y^2 = 6 \end{cases} \rightarrow \lambda = y$ if $x \neq 0$

$\begin{cases} x^2 = 4y^2 \\ x^2 + 2y^2 = 6 \end{cases} \rightarrow 4y^2 + 2y^2 = 6 \Rightarrow 6y^2 = 6 \Rightarrow y = \pm 1$

if $y = 1$ then $x = 2$ or -2
$y = -1$ then $x = 2$ or -2

so $(2,1)$ or $(-2,-1)$

$f(2,1) = 4 \cdot 1 = 4$
$f(-2,1) = 4 \cdot 1 = 4$

note $f(1,1) = 1$ so the value 4 is a maximum.