TMATH 126: Quiz 2

Key

You may use any work of yours that you made from last week. This includes, practice problems from the book and worked out WebAssign problems. This *does not* include photocopies of notes from the book or tutorials shown on WebAssign. You may also use a calculator, but you are not allowed to use any device that can access the internet.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   T F The equation $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$ represents a sphere with radius 3.

   2 T/F
   .5 try counterex/justify approp
   +1 got one that worked

   $x^2+8x \quad +y^2-6y \quad +z^2+2z \quad = -17$
   $+16 \quad +9 \quad +1 \qquad +16+9+1$ (complete the □ x3)

   $(x+4)^2 + (y-3)^2 + (z+1)^2 = 9$
   radius 3

   T F If the vectors $\vec{a}$ and $\vec{b}$ are parallel if and only if $\vec{a} \cdot \vec{b} = 0$.

   Recall $\dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|} = \cos\theta$ where θ is the angle between $\vec{a}$ & $\vec{b}$. Thus $\vec{a}\cdot\vec{b} = \|\vec{a}\|\|\vec{b}\| = \cos\theta$

   So $\vec{a}\cdot\vec{b} = 0$ if $\|\vec{a}\|\|\vec{b}\|\cos\theta = 0$ which could happen when $\theta = \pi/2$ or $\vec{a} \perp \vec{b}$.

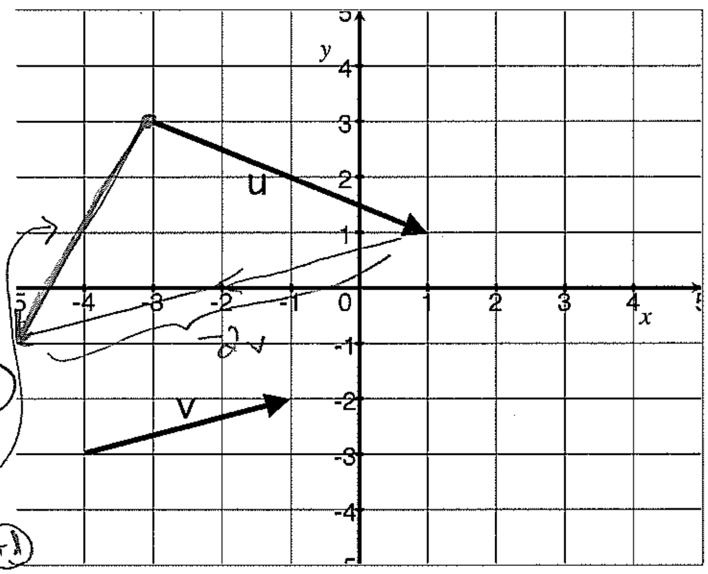
   T F Let $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$ then $(\vec{a} \cdot \vec{b}) \times \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})$.

   The first statement makes no sense $\underbrace{(\vec{a}\cdot\vec{b})}_{\text{scalar}/\#} \times \vec{c}$ and we can't cross a scalar with a vector.

   T F Let $\vec{a}, \vec{b} \in \mathbb{R}^3$ then $\|\vec{a} \times \vec{b}\| = \|\vec{b} \times \vec{a}\|$.

   The length of $\vec{a}\times\vec{b}$ is the area of a parallelogram with side lengths $\vec{a}$ and $\vec{b}$.
   The length of $\vec{b}\times\vec{a}$ is the area of a parallelogram with side lengths $\vec{b}$ and $\vec{a}$.
   Since a $\vec{a}$ by $\vec{b}$ parallelogram has the same area as a $\vec{b}$ by $\vec{a}$ parallelogram, the above equality holds.

2. The vectors $\vec{u}, \vec{v} \in \mathbb{R}^2$ and shown below, answer the following:



(a) [1] What are the components of $\vec{u}$?

$\langle 4, -2 \rangle$ or $4\vec{i} - 2\vec{j}$ (+.5) (+.5)

(b) [1] Find $||\vec{u}||$.

$\sqrt{2^2 + 4^2} = \sqrt{4+16}$ (+.5)

$= \sqrt{20} = 2\sqrt{5}$ (+.5)

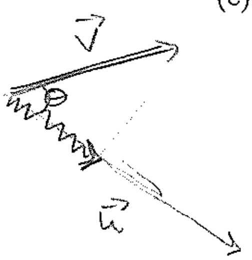
(c) [2] Draw the vector $\vec{u} - 2\vec{v}$. (+.5) (+.5)

note the components $\langle -2, -4 \rangle$ adding (+1)

(d) [2] Find $\vec{u} \cdot \vec{v}$.

$\underbrace{\langle 4, -2 \rangle \cdot \langle 3, 1 \rangle}_{\text{components (+.5)}} = \underbrace{4 \cdot 3 + -2 \cdot 1}_{(+1)} = 12 - 2 = \underbrace{10}_{(+.5)}$

(e) [4] Find the projection of $\vec{v}$ ~~only~~ onto $\vec{u}$



we want to find the shaded region shown on the left

Recall Sohcahtoa $\Rightarrow \cos\theta = \dfrac{||\text{~~~}||}{||\vec{v}||}$ (+.5) Recall $\cos\theta = \dfrac{\vec{v}}{||\vec{v}||} \cdot \dfrac{\vec{u}}{||\vec{u}||}$ (+.5)

length (+.5) $\Rightarrow ||\text{~~~}|| = ||\vec{v}|| \cos\theta$ ← alg/sub +.5

$= \cancel{||\vec{v}||} \dfrac{\vec{v}}{\cancel{||\vec{v}||}} \cdot \dfrac{\vec{u}}{||\vec{u}||} = \dfrac{\vec{v} \cdot \vec{u}}{||\vec{u}||} = \dfrac{10}{2\sqrt{5}}$ plug in +.5 (from (d)) (from (b)) $= \dfrac{5}{\sqrt{5}}$

if used formula but wrong way +1

$= \sqrt{5}$ +.5

(+1) [and the direction is $\dfrac{\vec{u}}{||\vec{u}||} = \dfrac{1}{2\sqrt{5}} \langle 4, -2 \rangle$ +.5 +.5 +.5 So $\dfrac{\sqrt{5}}{2\sqrt{5}} \langle 4, -2 \rangle = \langle 2, -1 \rangle$ put together (+.5)

3. [2] Find a *unit* vector orthogonal to both $\langle 1, -1, 1 \rangle$ and $\langle 0, 4, 4 \rangle$.

recall $\langle 1, -1, 1 \rangle \times \langle 0, 4, 4 \rangle$ (+.5) is orthogonal to both vectors given so

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 0 & 4 & 4 \end{vmatrix} = \vec{i}(-1 \cdot 4 - 1 \cdot 4) - \vec{j}(1 \cdot 4 - 0 \cdot 1) + \vec{k}(1 \cdot 4 - 0 \cdot -1) = -8\vec{i} - 4\vec{j} + 4\vec{k}$$ (+.5)

This vector does not have unit length though (+.5)

(it's length is $\sqrt{64+16+16} = \sqrt{64+32} = \sqrt{96}$) so we can

scale accordingly to get $\left\langle \dfrac{-8}{\sqrt{96}}, \dfrac{-4}{\sqrt{96}}, \dfrac{4}{\sqrt{96}} \right\rangle$ (+.5)