TMATH 126: Quiz 1

Key

You may use any work of yours that you made from last week. This includes, practice problems from the book and worked out WebAssign problems. This *does not* include photocopies of notes from the book or tutorials shown on WebAssign. You may also use a calculator, but you are not allowed to use any device that can access the internet.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

F (.5)
counter ex look (.5)
found counter ex (1)
notation/sense (.5)
stated (.5)

T F Let r be a number less than one, then the sequence $a_n = r^n$ converges.

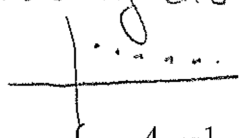
ex let $r = -1$

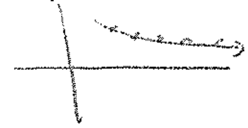
then $a_n = \{-1, 1, -1, 1, -1, \ldots\}$

T (.5)
stated (.5)
notation/sense (1)
reasonable explanation (1)

T F Given a sequence a_n, if there exists a function f, such that $f(n) = a_n$ for all positive integers n, then $\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x)$.

We're 'continuousifying' the sequence so instead of having discrete dots like on the left, we can work with a con't seq like on the right





2. Consider the sequence: $\left\{-3, \frac{4}{5}, \frac{-1}{5}, \frac{6}{125}, \frac{-7}{625} \cdots\right\}$.

   (a) [3] Find a formula for the n^{th} term where we start counting at one. (.5)

   $\left\{-3, \frac{4}{5}, \frac{-5}{25}, \frac{6}{125}, \frac{-7}{625}, \ldots\right\}$

   $\left\{\frac{-3}{5^0}, \frac{4}{5^1}, \frac{-5}{5^2}, \frac{6}{5^3}, \frac{-7}{5^4}, \ldots\right\}$

   patterns (.5)

   $a_n = \frac{(-1)^n (n+2)}{5^{n-1}}$

   n (.5)
   got num (.5
   powers (.5
   got den (.5

   (b) [1] Find the limit of the terms in the above sequence as $n \to \infty$.

   (.5)

   looks like zero since the den. is growing so quickly

   justification (.5)

   note $\left|\frac{(-1)^x(x+2)}{5^{x-1}}\right| = \frac{x+2}{5^{x-1}}$ so the dist from zero of each term is $\frac{x+2}{5^{x-1}}$. Note also $\lim_{x\to\infty} \frac{x+2}{5^{x-1}} \underset{L'H}{=} \lim_{x\to\infty} \frac{1}{(\ln 5) 5^{x-1}} = 0$

3. [5] Determine if the following sequences converge or diverge. If it converges, find the limit.

used Thm to connect with calc (+1)

$a_n = \frac{\ln(4n)}{\ln(8n)}$

$\lim_{n\to\infty} a_n = \lim_{x\to\infty} \frac{\ln(4x)}{\ln(8x)}$

(+.5) L'H

did L'Hospital (+1)

$= \lim_{x\to\infty} \frac{\frac{1}{4x}\cdot 4}{\frac{1}{8x}\cdot 8}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x}}$

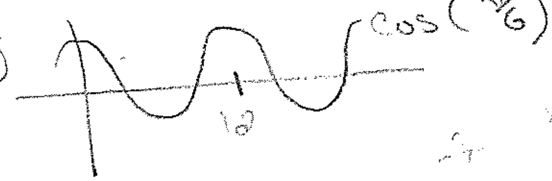
limits (+1)

$= \lim_{x\to\infty} 1 = 1$

$a_n = \cos\left(\frac{n\pi}{6}\right)$

$\lim_{n\to\infty} a_n = \lim_{x\to\infty} \cos\left(\frac{x\pi}{6}\right)$

reasoning (+1)

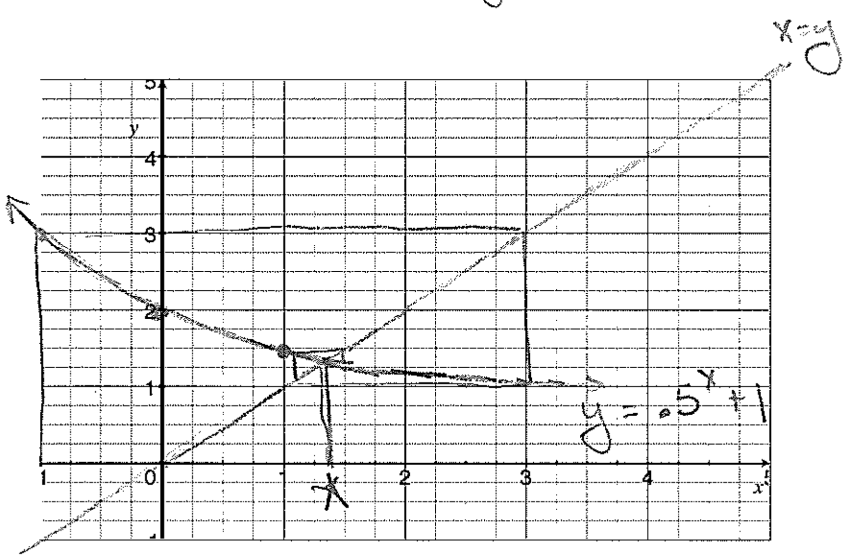


which has no limit

∴ diverges

got it (+.5)

4. Consider the recursively defined sequence $a_n = \frac{1}{2}^{a_{n-1}} + 1$.



(a) [1] If $a_1 = -1$, write down the first three terms of the sequence.

$\{-1, \frac{1}{2}^{-1}+1, \left(\frac{1}{2}\right)^{3}+1, \ldots\}$

$\{-1, 3, 9/8, \ldots$ (+.5) (+.5)

(b) [2] If $a_1 = -1$, does the sequence converge?
If the sequence does converge, identify the limit on the graph.

Yes to the value marked with a ✱. (+1)

cobwebbing (+1)

(c) [2] What values can a_1 be to guarantee that the sequence a_n will converge?

all choices of a_1 will converge to the value marked ✱ (+1.5)

stated (+.5)