TMATH 126: Quiz 1

You may use any work of yours that you made from last week. This includes, practice problems from the book and worked out WebAssign problems. This *does not* include photocopies of notes from the book or tutorials shown on WebAssign. You may also use a calculator, but you are not allowed to use any device that can access the internet.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

   T F Let r be a number less than one, then the sequence $a_n = r^n$ converges.

   T F Given a sequence a_n, if there exists a function f, such that $f(n) = a_n$ for all positive integers n, then $\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x)$.

2. Consider the sequence: $\left\{-3, \frac{4}{5}, \frac{-1}{5}, \frac{6}{125}, \frac{-7}{625} \cdots\right\}$.

   (a) [3] Find a formula for the n^{th} term where we start counting at one.

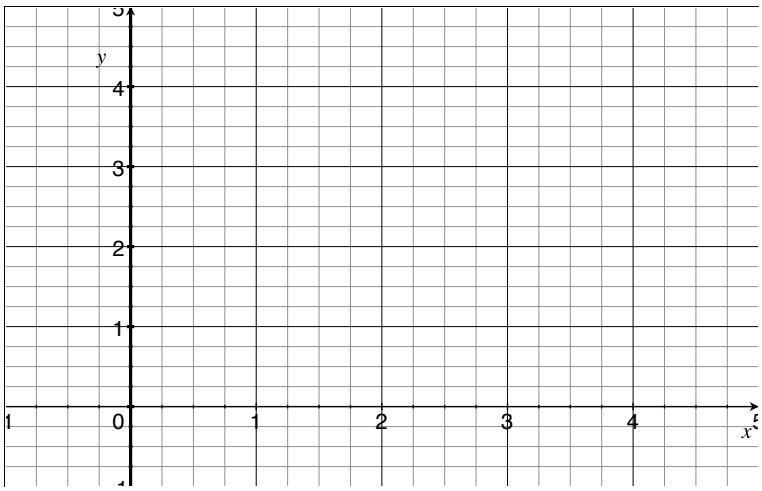
   (b) [1] Find the limit of the terms in the above sequence as $n \to \infty$.

3. [5] Determine if the following sequences converge or diverge. If it converges, find the limit.

$a_n = \dfrac{\ln(4n)}{\ln(8n)}$ $\qquad\qquad$ $a_n = \cos\left(\dfrac{n\pi}{6}\right)$

4. Consider the recursively defined sequence $a_n = \frac{1}{2}^{a_{n-1}} + 1$.

   (a) [1] If $a_1 = -1$, write down the first three terms of the sequence.



   (b) [2] If $a_1 = -1$, does the sequence converge?
   If the sequence does converge, identify the limit on the graph.

   (c) [2] What values can a_1 be to guarantee that the sequence a_n will converge?