Exam 2

Tmath 126

Practice

1. [12] TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) $(\overrightarrow{a} \cdot \overrightarrow{b}) \cdot \overrightarrow{c} = \overrightarrow{a} \cdot (\overrightarrow{b} \cdot \overrightarrow{c})$.

   (b) $||\overrightarrow{a} \times \overrightarrow{b}|| = ||\overrightarrow{b} \times \overrightarrow{a}||$.

   (c) If $f(x, y)$ is a continuous function, the first-order derivatives exist, and $f_x(0, 0) = 0 = f_y(0, 0)$, then f has a local minimum or maximum at the point $(0, 0)$.

   (d) If $f(x, y)$ is a continuous function, the first-order derivatives exist, and f has a local minimum or maximum at the point $(0, 0)$, then $f_x(0, 0) = 0 = f_y(0, 0)$.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the function g pictured to the right and answer the following multiple choice questions.

   (a) [1] $g_x(x_0, y_0)$ is

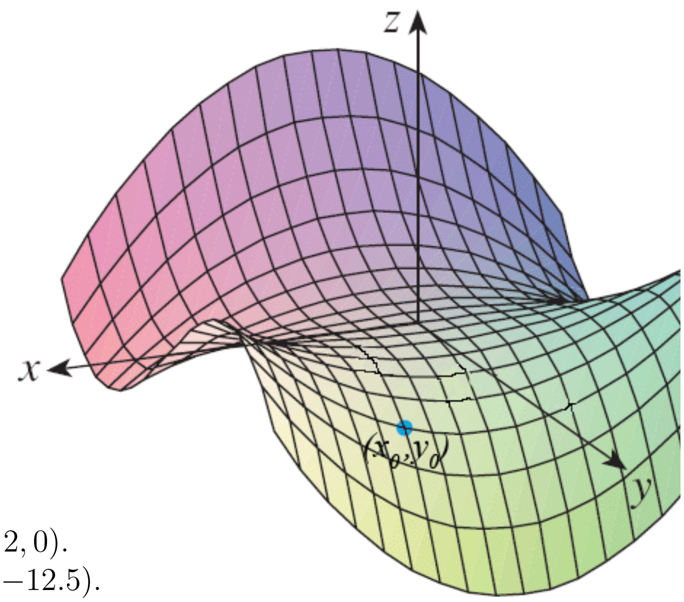
      i. > 0

      ii. < 0

   (b) [1] $\left.\dfrac{\partial g}{\partial y}\right|_{(x_0,y_0)}$ is

      i. > 0

      ii. < 0



3. Let $P(1, 3, 2)$, $Q(3, -1, 6)$, and $R(5, 2, 0)$. Also let $S(3, 6, 1.5)$ and $T(-9, -14, -12.5)$.

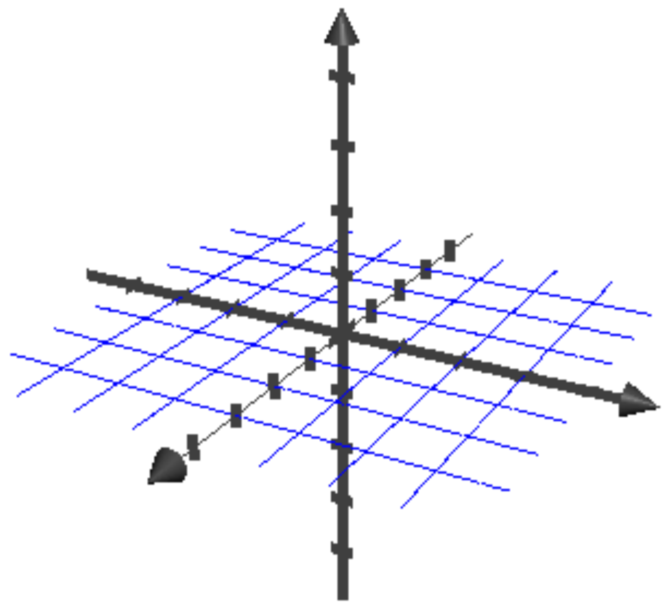
   (a) [3] Find the equation of the plane that passes through P, R, and Q.

   (b) [] What angle does the plane you found in part (a) intersect the xy-plane?

   (c) [2] Does the line that passes through S and T intersect the plane you found in part (a)? Justify yourself.

4. Consider the points: $P(1,-3,-2)$, $Q(2,0,-4)$, and $R(6,-2,-5)$.

   (a) [2] Plot the points P, Q, and R.



   (b) [3] Find the length of $\overrightarrow{PR}$

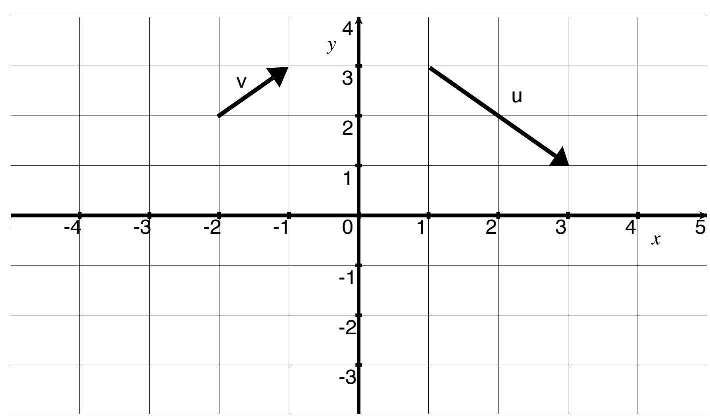
   (c) [4] Use calculus methods to determine of $\triangle PQR$ is a right triangle or not.

5. Consider the vector $\overrightarrow{v}$ and $\overrightarrow{u}$ shown to the right.

   (a) [1] Draw the vector $2\overrightarrow{u}$.

   (b) [1] Draw the vector $-3\overrightarrow{v}$.

   (c) [2] Draw the vector $2\overrightarrow{v}-\overrightarrow{u}$.



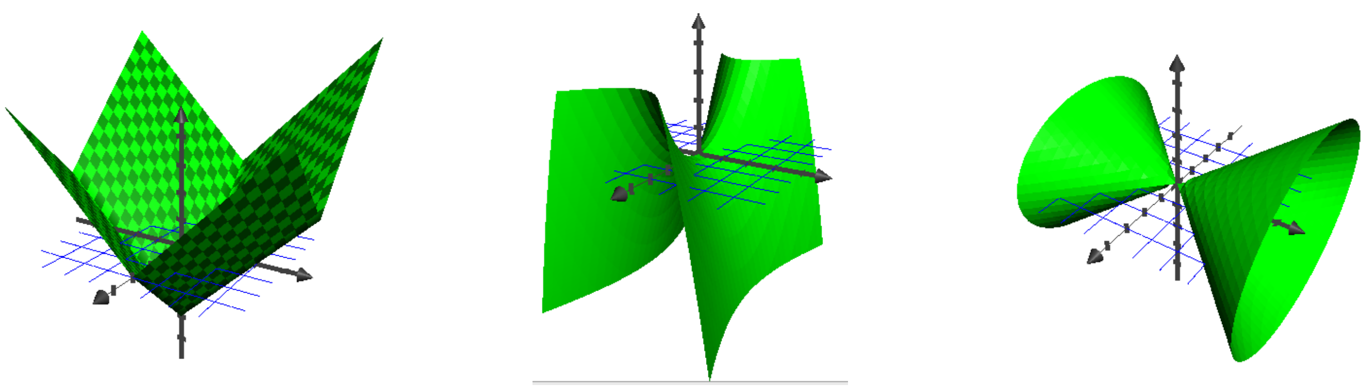
6. [3] Consider the equation $2z = \dfrac{x^2}{2} - 2y^2$.

(a) Does the above equation describe a function of x and y? Why or why not?

(b) Describe the contour curves of the graph of the equation above. That is, describe the intersection of the graph of the above equation with the planes $z = k$ where k is some constant.

(c) Describe the intersection of the graph of $2z = \dfrac{x^2}{2} - 2y^2$ with planes parallel to the xz axis. That is, when $y = k$ for some constant k.

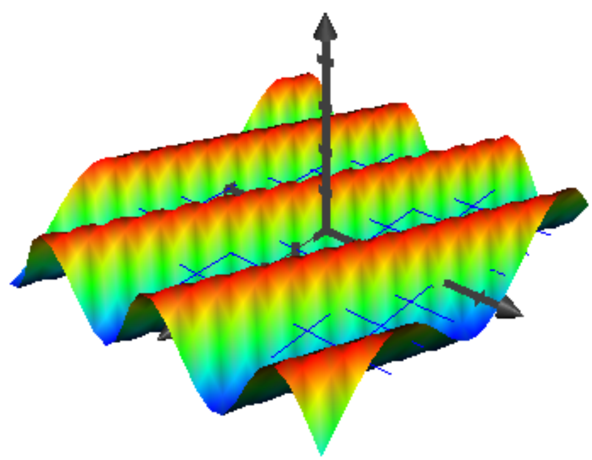
(d) Which (if any) of the following is a graph of the above function?



7. Find an equation of all points (x, y, z) twice the distance from the point $A(0, 0, 1)$ as it is from the point $B(0, 0, -1)$. You do not need to simplify.

8. Consider the function $f(x, y) = -\sin(x + 2y)$ for the following questions.

   (a) [3] Find the gradient of f .

   (b) [1] Evaluate the gradient at the point $(0, 0)$.

   (c) [2] Interpret your answer in (b) graphically and consider referencing the graph of f shown to the right.

   (d) [3] Find the linear approximation of f at the point $(0, 0)$.

9. Use Calculus methods to to find the (x, y, z) coordinates in $\mathbb{R}^3$ to find and classify the critical points of the function

$$f(x, y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4.$$

note: a graph of this function would normally be provided.