

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1.  TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample.

(a) If there exists some number  $M$  such that  $a_n \leq M$  for all  $n$ , then  $\{a_n\}$  converges.

(b) The Taylor series is an example of a power series.

(c) Given a function  $f$ , the associated Taylor series  $T$  has the property that  $f(x) = T(x)$  for all  $x$ .

(d)  $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] Write the following sum in expanded form:

$$\sum_{n=1}^4 \frac{\sqrt{2n+1}}{n!}$$

3. [4] Write the following sum using the sigma notation:

$$1 - \frac{2}{3} + \frac{3}{9} - \frac{4}{27} + \frac{5}{81}$$

4. [20] Compute the following if possible.

$$\lim_{n \rightarrow \infty} \frac{10^{n+1}}{9^n}$$

$$\sum_{n=1}^{\infty} \frac{(2e)^n}{6^{n-1}}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{2}{n}$$

5. Let  $p(x) = x^6 - 5x^2 + 3x - 2$ .

(a) [5] Find the second order Taylor polynomial  $T_2(x)$  based at  $b = 1$ .

(b) [5] Bound the error  $|p(x) - T_2(x)|$  on the interval  $[0.5, 1.5]$ .

6. [10] Find the Taylor series expansion for  $\frac{x}{4+x}$  centered at 0, and find out where it converges.

7. [10] Compute the following indefinite integral.

$$\int \frac{\sin(x)}{x} dx$$

8. [10] Use geometric series to show  $0.99999\dots = 1$ .