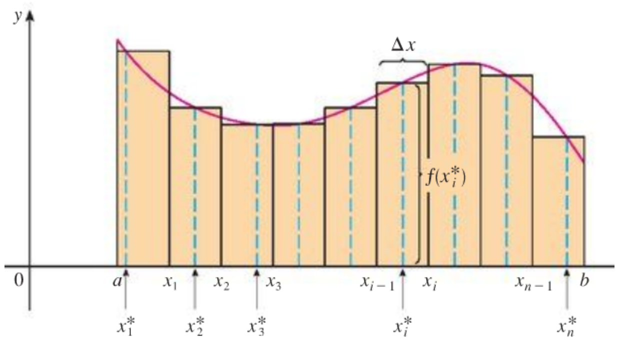
3D Integration

Recall the definition of the definite integral of a function of a single variable:

Let $f(x)$ be defined on $[a, b]$ and let $x_0, x_1, \ldots, x_n$ be a partition of $[a, b]$. For $i = 1, 2, \ldots, n$, let $x_i^* \in [x_{i-1}, x_i]$. Then



$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^n f(x_i^*)\Delta x.$

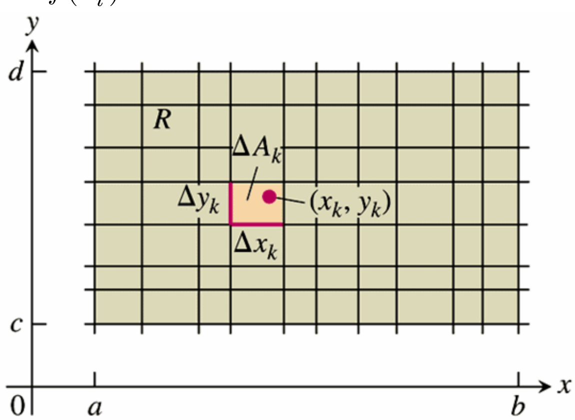
Generalizing from one variable to two.

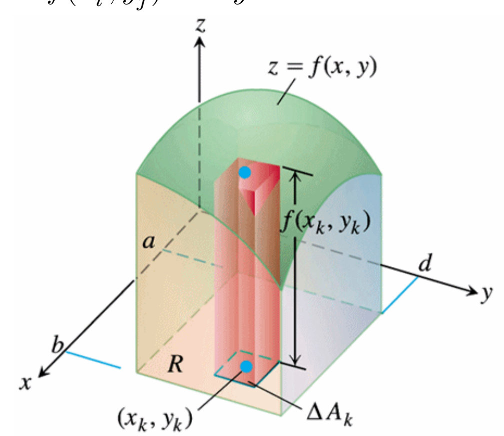
$f(x)$ on an interval $[a, b]$

$f(x_i^*)\Delta x$ little bit of area

$f(x, y)$ on rectangle $R = [a, b] \times [c, d] =$

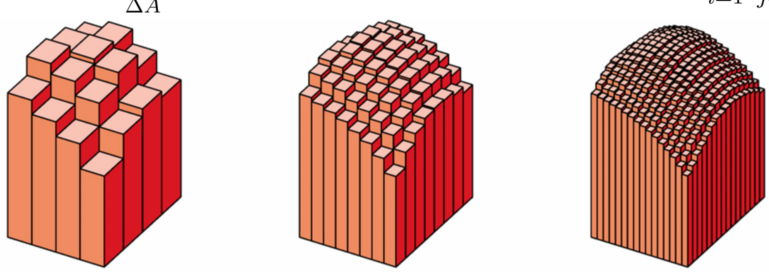
$f(x_i^*, y_j^*)\Delta x\Delta y$ little bit of volume





Double Riemann Sum If $f(x, y) \geq 0$ the double Riemann sum approximates the volume under the surface.

$$V \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i^*, y_j^*)\underbrace{\Delta x\Delta y}_{\Delta A} \qquad \iint_R f(x, y)dA = \lim_{\substack{n\to\infty \\ m\to\infty}} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i^*, y_j^*)\Delta A$$



(a) $n = 16$ (b) $n = 64$ (c) $n = 256$

1. Let R be the rectangle $1 \leq x \leq 1.2$ and $2 \leq y \leq 2.4$. If the values for $f(x, y)$ are as specified below, find a Riemann sum approximation for $\iint_R f(x,y)dA$ with $\Delta x = 0.1$ and $\Delta y = 0.2$. Assuming the function is reasonably "well behaved", determine if your sum is an over- or under-estimate.

$y \setminus x$	1.0	1.1	1.2
2.0	5	7	10
2.2	4	6	8
2.4	3	5	6

2. Evaluate the integrals

(a) $\displaystyle\int_0^3 \int_0^4 (4x + 3y)dx\ dy$

(b) $\displaystyle\int_0^4 \int_0^3 (4x + 3y)dy\ dx$

(c) $\displaystyle\int_1^3 \int_0^4 e^{x+y} dy\ dx$

(d) $\displaystyle\int_0^4 \int_1^3 e^{x+y} dx\ dy$