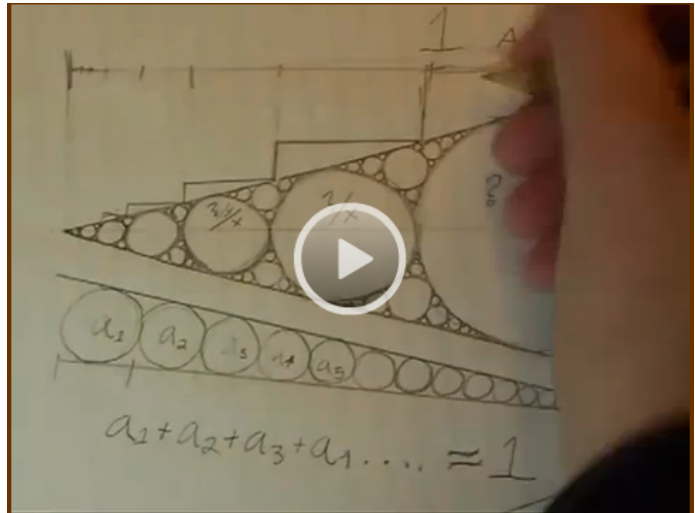


# Geometric Series

Adapted to complement ViHart's *Infinity Elephants* video.

1. Consider the sequence  $a_n = \frac{1}{3}^n$ .

- (a) Write down a few of the terms in the sequence  $a_n$ .



We will follow Vi Hart's lead and record the sequence of partial sums by moving horizontally across the page.

- (b) Draw a box (or an elephant) that is  $\frac{1}{3}$  units high &  $\frac{1}{3}$  units wide with the lower left "corner" on the origin.

The height will correspond with  $a_1$  and the total width across the page corresponds with  $s_1$ .

- (c) Draw the second box that has height  $a_2$  immediately to the right of the box (or elephant) you drew in (b) (that is with the lower left "corner" at the point  $(\frac{1}{3}, 0)$ ).

- (d) Identify the length that corresponds with  $s_2$ .

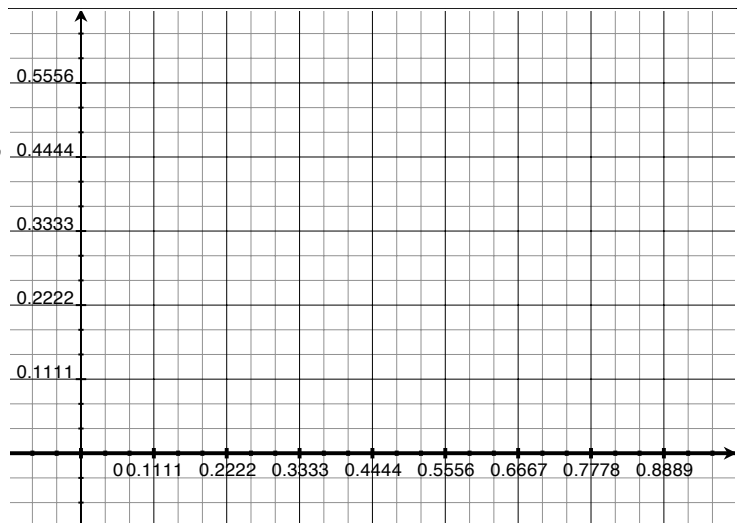
- (e) Draw a third box (or elephant) with height and width  $a_3$  immediately to the right of the box (or elephant) you drew in (c).

- (f) Identify the length that corresponds with  $s_3$ .

- (g) It gets difficult to repeat the procedure for  $s_4$ ,  $s_5$  and so on, but can you identify  $\lim_{n \rightarrow \infty} s_n$  is on the graph above?

- (h) Just as Vi Hart drew a line that skimmed across the tops of the circles, draw a line that skims the top right hand side of the boxes. Find the equation of the line you just drew.

- (i) Find  $\lim_{n \rightarrow \infty} s_n$



The above is an example of a geometric series. In general, a *geometric series* is an infinite series of the form:

$$a + ar + ar^2 + ar^3 + ar^4 + ar^5 + \dots + ar^{n-1} + \dots = \sum_{n=1}^{\infty} ar^{n-1}$$

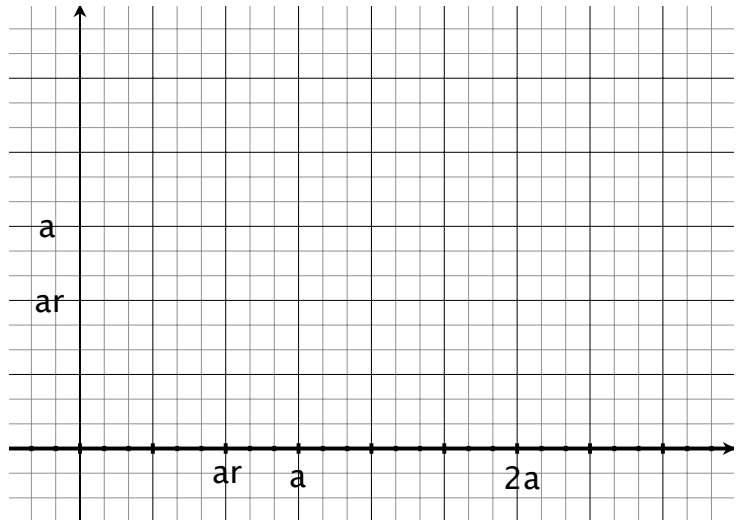
Or, using the definition of a series,  $\lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} (a + ar + ar^2 \dots + ar^n)$ .

2. Fix  $a$  and  $r$  as positive numbers with  $r < 1$ . We will mimic the steps done on the front side of this worksheet to find what the geometric series converges to in general. Consider the sequence  $a_n = ar^{n-1}$ .

- (a) Write down a few of the terms in the sequence  $a_n$ .

- (b) Draw a box (or an elephant) that is  $a_1$  units high &  $a_1$  units wide with the lower left “corner” on the origin.

The height will correspond with  $a_1$  and the total width across the page corresponds with  $s_1$ .



- (c) Draw the second box that has height  $a_2$  immediately to the right of the box (or elephant) you drew in (b) (that is with the lower left “corner” at the point  $(a,0)$ ). Notice to see the picture better I’ve labeled what  $ar$  is (effectively making  $r$  equal  $\frac{2}{3}$ , but by keeping the  $r$ ’s in the picture, you’ll be able to keep the general result).
- (d) Identify the length that corresponds with  $s_2$ .
- (e) Draw a third box (or elephant) immediately to the right of the box (or elephant) you drew in (c).
- (f) Identify the length that corresponds with  $s_3$ .
- (g) It gets difficult to repeat the procedure for  $s_4$ ,  $s_5$  and so on, but can you identify  $\lim_{n \rightarrow \infty} s_n$  is on the graph above?
- (h) Just as Vi Hart drew a line that skimmed across the tops of the circles, draw a line that skims the top right hand side of the boxes. Find the equation of the line you just drew.

- (i) Find  $\lim_{n \rightarrow \infty} s_n$