Exam 2

Tmath 126

Summer 2011

1. [12] TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{b} \cdot \overrightarrow{a}$.

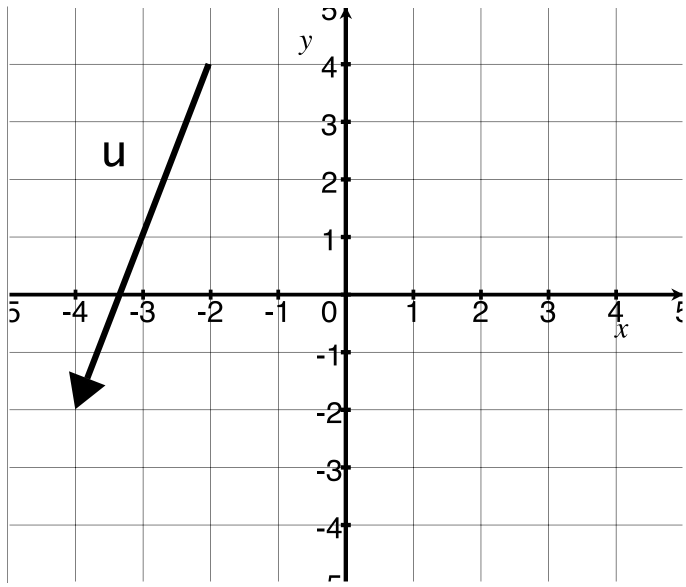
   (b) $(\overrightarrow{a} \times \overrightarrow{b}) \cdot \overrightarrow{a} = 0$.

   (c) Two lines parallel to a plane are parallel.

   (d) If $f(x, y)$ is a continuous function, the first-order derivatives exist, and $f_x(0,0) = 0 = f_y(0,0)$, then f has a local minimum or maximum at the point $(0,0)$.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. The vector $\overrightarrow{u} \in \mathbb{R}^2$ and shown below, answer the following:



(a) [1] What are the components of $\overrightarrow{u}$?

(b) [1] Draw the vector $\overrightarrow{v} = \langle 1, -3 \rangle$.

(c) [1] Find $||\overrightarrow{u}||$.

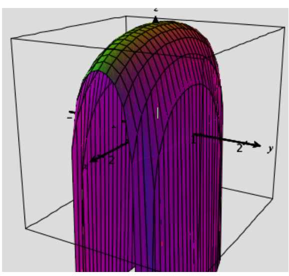
(d) [2] Draw the vector $\overrightarrow{u} + 3\overrightarrow{i}$.

(e) [2] Find the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$.

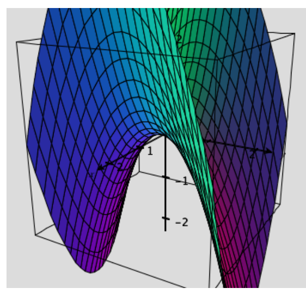
(f) [4] Find the projection of $\overrightarrow{v}$ onto $\overrightarrow{u}$.

3. Find the equation of the plane that passes through the points $(0, 1, 1)$, $(1, 0, 1)$, and $(1, 1, 0)$.

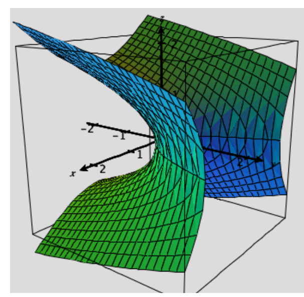
i)



ii)



iii)



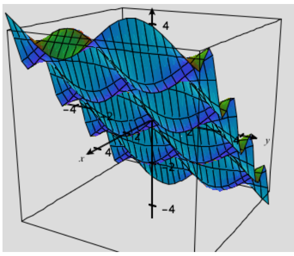
4. Consider the three graphs above for the following questions:

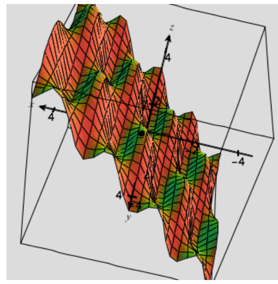
   (a) [3] Match the following equations to their respective graphs:
   A. $z = x^2 - y^2$ B. $y = x^2 - z^2$ C. $z = \ln(9 - x^2 - 9y^2)$

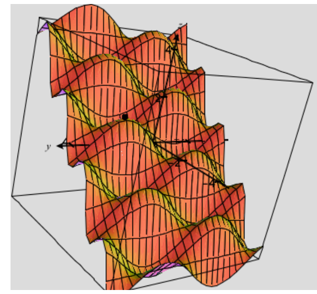
   (b) [1] Identify which of the above are graphs of functions.

   (c) [2] For each of the expressions above that are functions, identify the domain and the range.

5. Three views of the function $f(x, y) = x + \cos(3x)\sin(y)$ are shown below and may be used for the following questions. The point $(0, \frac{\pi}{2}, f\left(0, \frac{\pi}{2}\right))$ is identified on the graph.







(a) [3] Find the gradient of f .

(b) [1] Evaluate the gradient at the point $(0, \frac{\pi}{2})$.

(c) [1] Interpret your answer in (b) graphically and consider referencing the graph of f shown to the right.

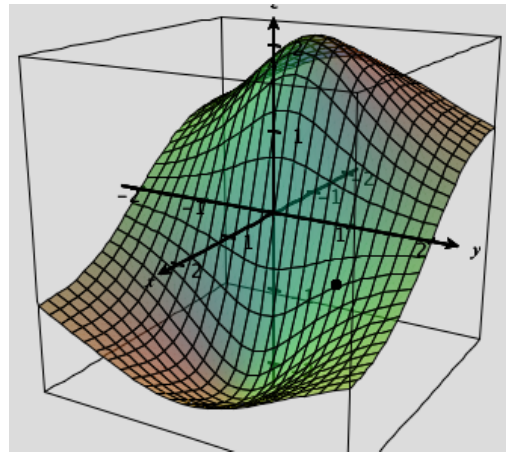
(d) [3] Find the linear approximation of f at the point $(0, \frac{\pi}{2})$.

6. [3] Consider the graph of f shown to the right with the point $f(\frac{1}{3}, 1)$ identified.

   (a) Determine if $f_x(\frac{1}{3}, 1)$ is positive or negative.

   (b) Determine if $f_y(\frac{1}{3}, 1)$ is positive or negative.

   (c) Explain how you know.



7. [7] Consider the function $g(x, y) = 3(x^2 + y^2)e^{y^2 - x^2} - 2$. Three views of the function g are shown below. Identify all critical points and then classify them as local minimums, local maximums, or saddle points.