Exam 1 | **Tmath 126** | Summer 2011

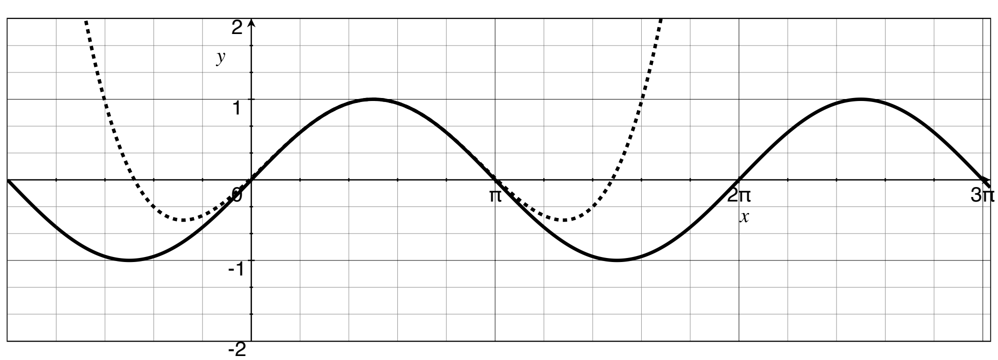
1. [8] TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   (a) A sequence can be thought as an "infinite sum".

   (b) Let $r = -.7$, then the sequence $a_n = r^n$ converges.

   (c) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

   (d) The dotted function below is the 4^{th} Taylor polynomial of $\sin(x)$ centered at 0.



Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] Write the following sum using the sigma notation:

$$-3 + \frac{4}{5} + \frac{-1}{5} + \frac{6}{125} + \frac{-7}{625}$$

3. [12] Compute the following if possible.

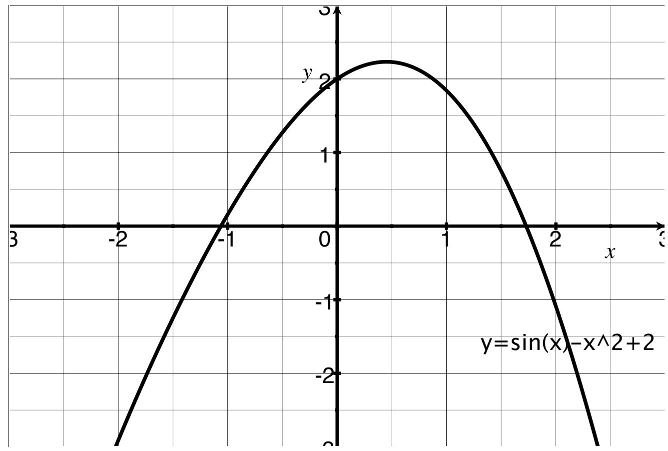
$$\lim_{n\to\infty} \tan\left(\frac{4n\pi}{3+12n}\right)$$

$$\sum_{n=1}^{\infty} \frac{3n^2-3}{n^2+4}$$

$$\sum_{n=1}^{\infty} 9(0.4)^{n-1}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$$

4. Below is the graph of the function $f(x) = \sin(x) - x^2 + 2$.

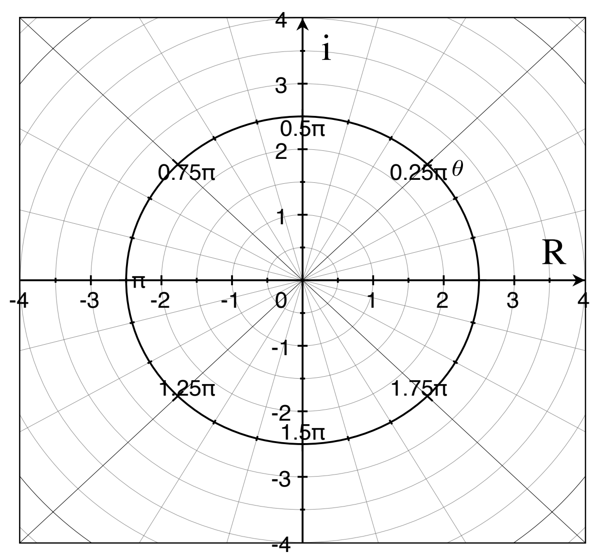


   (a) [3] Find the equation the line tangent to the graph of f when $x = 0$.

   (b) [2] If Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 0$, find the second approximation x_2.

5. Let P and Q be complex numbers where $P = e^{\frac{i\pi}{6}}$ and $Q = -2 + (2\sqrt{3})i$

   (a) [1] Convert P to rectangular coordinates (that is, of the form $a + bi$).



   (b) [1] Convert Q to polar form (that is, of the form $re^{i\theta}$).

   (c) [1] Plot P and Q on the complex plane provided to the right.

   (d) [1] Multiply $P \cdot Q$.

   (e) [2] Multiply P^{20} and then plot this point on the axes provided on the right.

6. Consider the function $\dfrac{2}{3-x}$.

(a) [3] Find a power series representation for the above function.

(b) [3] Find the second order Taylor polynomial $T_2(x)$ based at $b = 0$.

(c) [4] Bound the error $|f(x) - T_2(x)|$ on the interval $[-1.5, 1.5]$.