Key

TMATH 126: Quiz 4

You may use:

- any kind of calculator that cannot access the internet and
- a one-sided 3 × 5" card for this quiz.

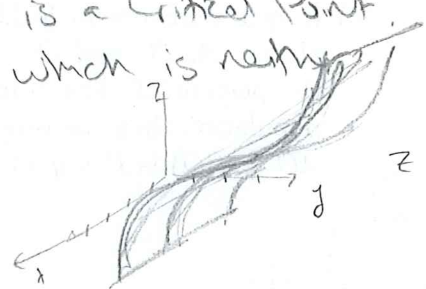
Show *all* your supporting work (numerically, algebraically, or geometrically) for each and simplify. *No credit* is given without supporting work.

1. [3] TRUE/FALSE: Circle T if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample or brief justification.

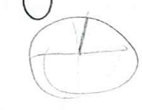
   T F If f is a function such that $f_x(4,-2) = 0$ and $f_y(4,-2) = 0$ then $f(4,-2)$ is a maximum or a miniumum.

start (+.5)
answer (+1)
true (+.5)

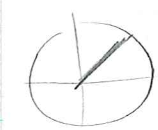
Just means $(4,-2,f(4,-2))$ is a Critical Point
(+1) This could be a saddle which is neither
ex $z = (x-4)^3$



$z = (y+2)^3$



2. Let $f(x,y) = \tan\left(\frac{\pi}{4}x\right) + \frac{1}{y}$



   (a) [5] (TangentWks) Find the local linearization (or linear approximation) of $f(x,y)$ when $x = y = 1$.

(+.5) ie find the plane tangent to f

(+1.5) $f_x(x,y) = \frac{\pi}{4}\sec^2\left(\frac{\pi}{4}x\right)$, $f_y(x,y) = \frac{-1}{y^2}$

(+1) $m_x = \frac{\pi}{4}\sec^2\left(\frac{\pi}{4}\right) = \frac{\pi}{4}(\sqrt{2})^2 = \frac{\pi}{2}$, $m_y = -1$

(+1) thru the point $f(1,1) = \tan\left(\frac{\pi}{4}\right) + 1 = 1 + 1 = 2$

(+1) $z - (2) = \frac{\pi}{2}(x-1) - 1(y-1)$

If use $f(x,y) = \tan\left(\frac{\pi}{3}x\right) + \frac{1}{y}$

(+1.5) $f_x(x,y) = \frac{\pi}{3}\sec^2\left(\frac{\pi}{3}x\right)$, $f_y(x,y) = \frac{-1}{y^2}$

(+1) $m_x = \frac{4\pi}{3}$, $m_y = -1$

OR

(+1) thru the point $(1,1,\sqrt{3}+1)$

(+1) $z - (\sqrt{3}+1) = \frac{4\pi}{3}(x-1) - 1(y-1)$

like (+.5)

   (b) [3] (HW8 §13.4 #2) Note that f intersects the plane below when $x = y = 1$. Determine if the f is tangent or perpendicular to the plane that this intersection point. $\langle -4\pi, 3, 3\rangle \cdot \left[\langle x,y,z\rangle - \langle 1,1,1+\sqrt{3}\rangle\right] = 0$

angles/dot product (+1.5)
completion/alg (+1.5)

note $f(1,1) = \frac{1}{\sqrt{2}} + 1$ but $\langle -4\pi,3,3\rangle \cdot \left[\langle 1,1,\frac{1}{\sqrt{2}}+1\rangle - \langle 1,1,1+\sqrt{3}\rangle\right] = \langle -4\pi,3,3\rangle \cdot \langle 0,0,\frac{1}{\sqrt{2}}-\sqrt{3}\rangle$

$= 0 + 0 + 3\left(\frac{1}{\sqrt{2}}-\sqrt{3}\right) \neq 0$ so not an intersection.

$z - \left(\frac{1}{\sqrt{2}}+1\right) = \frac{\pi}{2}(x-1) - 1(y-1)$

$0 = \frac{\pi}{2}(x-1) - (y-1) - \left(z - \left(\frac{1}{\sqrt{2}}+1\right)\right)$

$0 = \langle \frac{\pi}{2}, -1, -1\rangle \cdot \left(\langle x,y,z\rangle - \langle 1,1,\frac{1}{\sqrt{2}}+1\rangle\right)$

Note that $\langle \frac{\pi}{2},-1,-1\rangle \cdot \langle -4\pi,3,3\rangle / \left(\sqrt{\frac{\pi^2}{4}+1+1}\sqrt{16\pi^2+9+9}\right) \neq 0, \neq 1$

so neither

$z - (\sqrt{3}+1) = \frac{4\pi}{3}(x-1) - (y-1)$

$0 = \frac{4\pi}{3}(x-1) - (y-1) - (z - (\sqrt{3}+1))$

$0 = \langle \frac{4\pi}{3}, -1, -1\rangle \cdot \left(\langle x,y,z\rangle - \langle 1,1,\sqrt{3}+1\rangle\right)$

// to the plane b/c scalar of given?

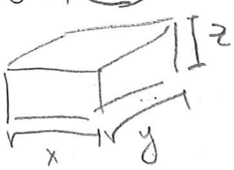
3. [9] Choose *ONE* of the following. Clearly identify which of the two (a or b) you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.
For either situation a xor situation b, you need to:

- identify the function you need to optimize and
- outline the steps needed to find the maximums and verify this is the maximum. Make sure to highlight any steps involving calculus or precalculus techniques. You do not need to perform the steps but you do need to make sure that your process works.

(a) (WebHW15 #6) The material for constructing the base of a rectangular box costs three times as much per unit area as the material for constructing the sides. We would like to maximize the volume for a fixed cost.

(b) (SuggestedProblem §13.9 #15) Common blood types are determined by three alleles, A, B, and O. If p is the percent of allele A in the population, q is the percent of allele b in the population and r is the percent of allele O in the population then the proportion of individuals with a mixed blood type (e.g. AB, AO or BO) is $P(p, q, r) = 2pq + 2pr + 2qr$. We would like to maximize P.

(a) Start (1.5)



(+1)

(1.5) $\text{Cost} = 3xy + 2xz + 2yz$

(+1) $\text{Volume} = x \cdot y \cdot z$ ← want to maximize

(+2) Need to reduce the Volume function to just a function of 2 variables
→ Solve for x in Cost function
→ Sub this in to the Volume function

(+2) Find the Critical Points (CP)
→ find y & z so that $\partial V/\partial x = 0$ AND $\partial V/\partial y = 0$.
Newton's Method may be needed.

(+1) Use the 2nd Derivative test on each (CP) to determine if we have a max or min

(+1) Notation/clear

(b) Start +.5

Note that A, B & O are the complete list of possible alleles

+.5 ⇒ $p + q + b = 1$ (*)

$P(p,q,r) = 2pq + 2pr + 2qr$ ← want to maximize (+1)

(+2) Need to reduce P's function to just a function of 2 variables
→ Solve for p in *'d equation
→ Sub this into P's equation

(1.25) Find the Critical Points (CP)
→ find q & r so that $\partial P/\partial q = 0$ AND $\partial P/\partial r = 0$.
Newton's Method may be needed

(+1) Use the 2nd Derivative test on each CP to determine if we have a max or min

(+1) notation/clear

Note: #3 could have been completed w/ Lagrange Multipliers instead.