Key

TMATH 126: Quiz 3

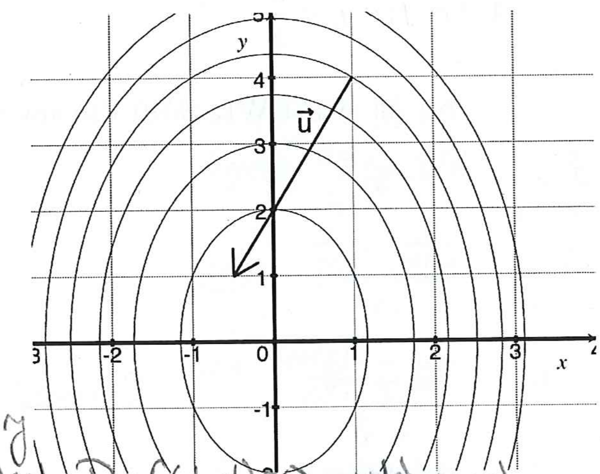
You may use:

- any kind of calculator that cannot access the internet and
- a one-sided 3 × 5" card for this quiz.

Show *all* your supporting work (numerically, algebraically, or geometrically) for each and simplify. *No credit* is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample or brief justification.

T (F) Let $\vec{u}$ be the vector shown on the contour graph to the right. Then $D_{\vec{u}} f(1,4)$ is zero.



Stat (.5)
answer (+1)
true (.5)
justify (+1)

The arrow is passing several different level curves meaning it is either going up or down. Certainly it is not remaining at the same level which is what $D_{\vec{u}} f(1,4) = 0$ would imply.

(T) F (WebHW13 #1) We have to use the product rule to find $\frac{\partial}{\partial x}(x\cos(xy))$

We treat y as a constant much like $x\cos(7x)$. In this ex, we would still have to use the product rule b/c it's x times $\cos(xy)$ (also involving an x)

2. [4] (Suggested §13.3 #29) Let $f(x,y) = x\cos(xy)$, find $\frac{\partial f}{\partial y}$.

$$\frac{\partial}{\partial y}\left(x\cos(xy)\right) = -x\sin(xy)\underbrace{\frac{\partial}{\partial y}(xy)}_{\text{(Chain rule)}} = -x^2\sin(xy)$$

(much like $\frac{\partial}{\partial y}(7\cos(7y)) = -7\cdot\sin(7y)\cdot(7y)' = -7^2\sin(7y)$)

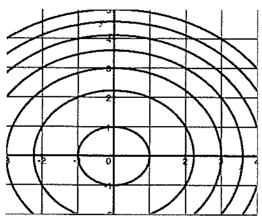
fix x (+1)
Chain (+1)
der of cos (+1)
got it (+1)

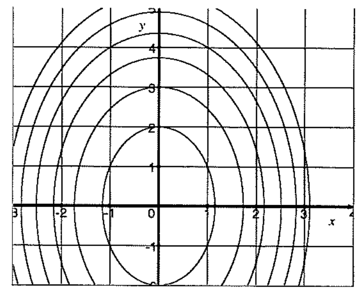
(+2)

3. [3] (3D Function Wks #2) Match the function with it's level curves. Justify your answer. (+1)

$a(x,y) = 5000 - 30x^2 - 10y^2$ concentric ovals

$b(x,y) = x^2 + y^2$ concentric circles





4. Let $f(x,y) = \dfrac{x}{y}$.

(a) [3] (WebHW12 #19) Use any method to find $\lim\limits_{h\to 0} \dfrac{f(x,y+h)-f(x,y)}{h}$.

$$\lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x}{y+h} - \frac{x}{y}}{h} \quad (+1)$$

$$= \lim_{h\to 0} \frac{xy - x(y+h)}{y(y+h)} \div h \quad \text{fractions (+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{xy} - \cancel{xy} - xh}{y(y+h)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-x\cancel{h}}{\cancel{h}y(y+h)} \quad \text{alg (+1)}$$

$$= \lim_{h\to 0} \frac{-x}{y(y+h)} = \frac{-x}{y(y+0)}$$

$$= \frac{-x}{y^2}$$

or (+1) this is the definition of $\partial f/\partial y$

(+1) So we contract x as a constant (like $f(7,y) = 7/y$) and differentiate with respect to y ($f'(7,y) = (7y^{-1})' = -7y^{-2}$)

So $\frac{\partial}{\partial y}\left(\frac{x}{y}\right) = \frac{\partial}{\partial y}(xy^{-1})$

(+1) $= x(-1)y^{-2} = -xy^{-2}$ or $\frac{-x}{y^2}$.

(b) [4] (WrittenHW7 §13.6 #1) Find $D_{\vec{u}}f(1,1)$ where $\vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$.

note $\|\vec{u}\| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9+16}{25}} = 1$

$$D_{\vec{u}}f(1,1) = \nabla f(1,1) \cdot \langle 3/5, 4/5 \rangle \quad (+1)$$

$$= \left\langle \frac{1}{1}, \frac{-1}{1^2} \right\rangle \cdot \langle 3/5, 4/5 \rangle \quad (+1)$$

$$= \langle 1, -1 \rangle \cdot \langle 3/5, 4/5 \rangle \quad (+1)$$

$$= 3/5 + -4/5 = -1/5$$

note $\nabla f = \left\langle \frac{1}{y}, \frac{-x}{y^2} \right\rangle$ (+1)