TMATH 126: Quiz 2

Key

You may use:

- any kind of calculator that cannot access the internet and
- a one-sided 3 × 5" card for this quiz.

Show *all* your supporting work (numerically, algebraically, or geometrically) for each and simplify. *No credit* is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample or brief justification.

start (+.5)
answer (+1)
true (+.5)
justify (+1)

T F If $\vec{v}$ and $\vec{w}$ are 3 dimensional vectors, then $(\vec{v} \times \vec{w}) + (\vec{v} \cdot \vec{w})$ returns a vector.

$\vec{v} \cdot \vec{w}$ returns a scalar whereas $\vec{v} \times \vec{w}$ returns a vector.
We have no way of adding a vector to a scalar.

T F The set of (x, y, z) defined by $\langle x, y, z\rangle = \langle 6, -3, 1\rangle t + \langle 0, 0, 5\rangle$ where $t \in \mathbb{R}$ form a line.

$\langle 6,-3,1\rangle$ provides a direction to extend from whereas $\langle 0,0,5\rangle$ provides a point in space to start at

-or- false b/c $\langle x,y,z\rangle$ is a vector not points

2. [3] (Suggested §11.3 #13) Find the angle between the vectors $\vec{u} = \langle 1, 1, 1\rangle$ and $\vec{v} = \langle 2, 1, -1\rangle$. notation (+.5)

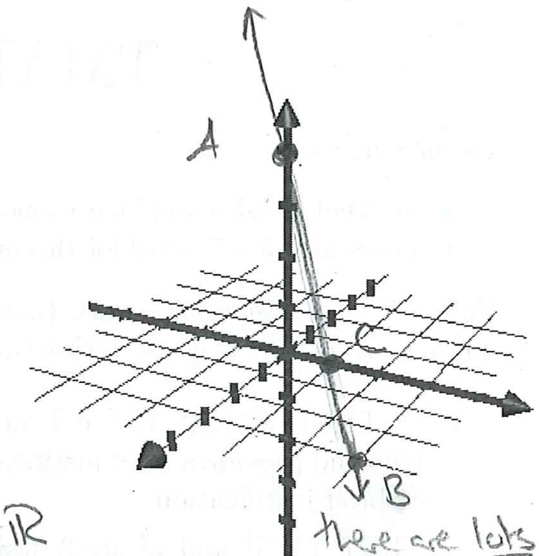
(+1) Recall $\dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|} = \cos\theta$ where θ is the angle between $\vec{u}$ & $\vec{v}$

$$\cos\theta = \frac{1\cdot 2 + 1\cdot 1 + 1(-1)}{\sqrt{1^2+1^2+1^2}\,\sqrt{2^2+1^2+(-1)^2}} \;(+.5)(+.5) = \frac{2+\cancel{1}-\cancel{1}}{\sqrt{3}\,\sqrt{6}} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$

So $\theta = \arccos\left(\sqrt{2}/3\right)$ solve (+.5)

$\approx 61.87°$ or 1.08 rad.

3. Consider the points $A(0,0,4)$, $B(3,3,0)$, and $C(0,1,0)$.



(a) [1] Find the components of $\overrightarrow{BA}$.

$\langle 0-3, 0-3, 4-0 \rangle$

(+.5) $\langle -3, -3, 4 \rangle$

(+.5) terminal point − initial point

(b) [3] (WebHW8 #2) Find an equation for the line passing through A and B.

$\langle x,y,z\rangle = \langle 3,3,-4\rangle t + \langle 3,3,0\rangle$ where $t \in \mathbb{R}$

$\langle x,y,z\rangle = \langle -3,-3,+4\rangle t + \langle 0,0,4\rangle$ where $t \in \mathbb{R}$

$\langle x,y,z\rangle = -3t, -3t, +4t+4$ where $t \in \mathbb{R}$

$x = -3t,\ y = -3t,\ z = +4t + 4$ where $t \in \mathbb{R}$

$\frac{x}{-3} = \frac{y}{-3} = \frac{z-4}{4}$

there are lots of answers for (b)

a line (+1)

through points (+1)

got line (+1)

(c) [4] (Dot&Cross Wks #3) Find the area of a triangle defined by A, B, and C.

(+.5) Note $\overrightarrow{BA} = \langle -3,-3,4\rangle$ and $\overrightarrow{BC} = \langle -3,-2,0\rangle$

(+1) Recall $\|\overrightarrow{BA} \times \overrightarrow{BC}\|$ is the area of the parallelogram formed by $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

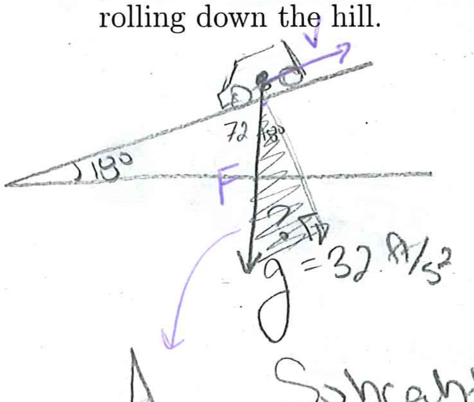
(+.5) Thus Area of $\triangle ABC = \frac{1}{2}\|\overrightarrow{BA} \times \overrightarrow{BC}\|$

compute cross (+1.5) & norm

$$\text{Area} = \frac{1}{2}\left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -3 & +4 \\ -3 & -2 & 0 \end{vmatrix} \right\| = \frac{1}{2}\left\|(0-8)\vec{i} - (0-12)\vec{j} + (+6-9)\vec{k}\right\| = \frac{1}{2}\sqrt{8^2+12^2+3^2} \approx 7.4$$

got it (+.5)

4. [3] (WebHW7 #5) A 5400 pound SUV (large car) is parked on an 18° slope. Assume the only force to overcome is gravity. Find the force required to keep the SUV from rolling down the hill.



picture (+.5)

understand what is wanted (+.5)

$\vec{F} = m \cdot \vec{a} = 5400\,\text{lb} \cdot 32\ \text{ft}/\text{s}^2$ (+.5)

$= 172800\vec{j}$

or Find $\vec{F}$ projected onto $\vec{v}$

$\vec{v} = (\cos 18°)\vec{i} + (\sin 18°)\vec{j}$ note $\|\vec{v}\| = 1$

90
18
72

(+.5)

Sohcahtoa

$\sin 18° = \frac{?}{5400 \cdot 32}$

(+.5)

$\Rightarrow ? = 5400 \cdot 32\ \text{lb}\,\frac{\text{ft}}{\text{s}^2} \cdot \sin 18\,\vec{v}$

$\approx 53{,}398\,\vec{v}$

got it (+.5)