TMATH 126: Quiz 1 /20

Key

You may use:

- any kind of calculator that cannot access the internet and
- a one-sided 3 × 5" card for this quiz.

Show *all* your supporting work (numerically, algebraically, or geometrically) for each and simplify. *No credit* is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample or brief justification.

   T (F) The series $\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$ converges.

   → note: problem b/c not defined at n=1; consider $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$

   Since the numbers continue to get larger (b/c $\lim_{n\to\infty} \frac{n}{\ln(n)} = \infty$) the infinite sum must as well.

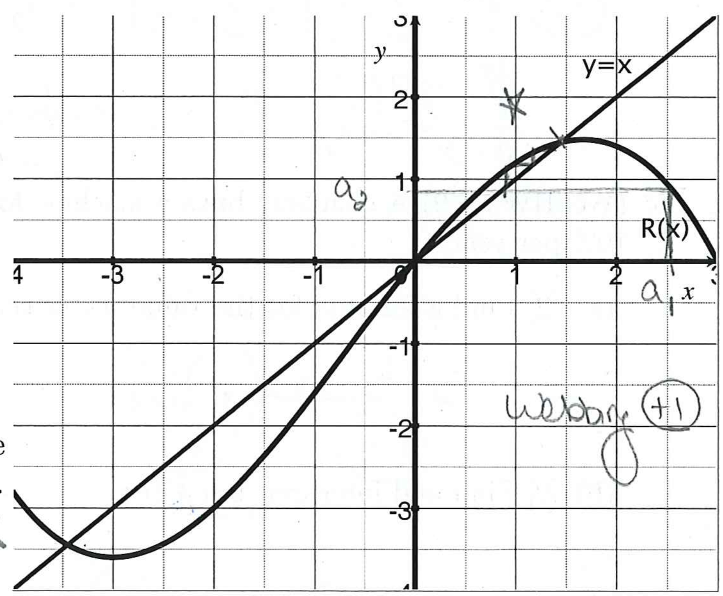
   note $\lim_{n\to\infty} \frac{n}{\ln(n)}$ "$\frac{\infty}{\infty}$" $\overset{LH}{=} \lim_{n\to\infty} \frac{1}{1/n} = \lim_{n\to\infty} n$ which diverges.

   state (+.5) answer (+1) true (+.5) justify (+1)

   (T) F If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n$ converges.

   If an infinite sum converges to a finite # we must be adding either zeros or #'s very close to zero, so in fact $\lim_{n\to\infty} a_n = 0$.

2. The graph of $R(x)$ and $y = x$ are both graphed to the right. Consider the recursively defined sequence where $a_n = R(a_{n-1})$ and $a_1 = 2.5$.



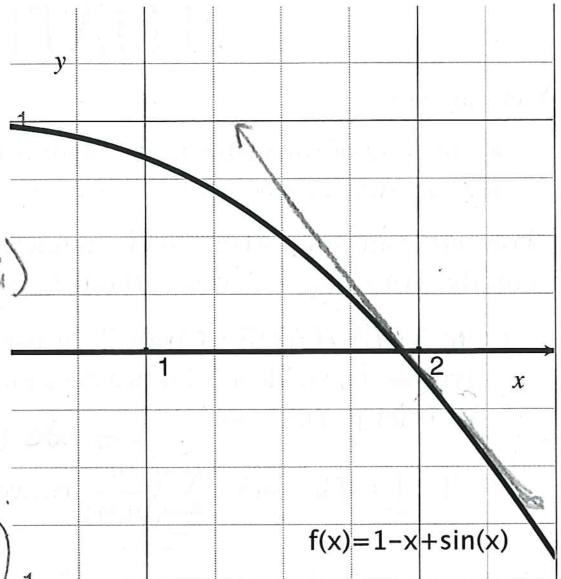
   (a) [1] (SequenceWks #1) Use the graph to estimate a_2.

   $\approx .3$

   (b) [2] (WrittenHW1§9.1 #3) Use the graph to estimate $\lim_{n\to\infty} a_n$.

   Converges to x coord. of ✱ ≈ 1.4 (+1)

3. (Suggested Problem §3.8 #15)
Consider $f(x) = 1 - x + \sin(x)$.



(a) [3] Find an equation of the line tangent to the graph of f at $x = 2$.

(+.5) looking for $y = mx + b$ or $y - y_1 = m(x - x_1)$

(+.5) $m = f'(2)$ $\quad f'(x) = -1 + \cos(x)$ (+.5)

$= -1 + \cos(2)$

≈ -1.416 (+.5)

(+.5) passes thru $(2, f(2)) = (2, 1 - 2 + \sin(2))$

$= (2, -1 + \sin(2))$

So $y - (-1 + \sin(2)) = (-1 + \cos(2))(x - 2)$ (+.5) or $\approx y - .0907 = -1.416(x - 2)$

(b) [2] Use part (a) to find the next approximation of the root shown in the graph when the initial guess is 2.

i.e. where does the tangent line hit the x-axis?

i.e. set $y = 0$ & solve for x

$0 - (-1 + \sin(2)) = (-1 + \cos(2))(x - 2)$ $\quad$ so $\dfrac{1 - \sin(2)}{-1 + \cos(2)} = x - 2$

$x = 2 + \dfrac{1 - \sin(2)}{-1 + \cos(2)} \approx 1.936$

4. [2] (WebHW3 #4) Consider the series $9 - 3 + 1 - \frac{1}{3} + \ldots$.
Determine if the series converges or diverges. If the series converges, find its limit.

(+.5) $3^2 - 3^2\left(\frac{1}{3}\right) + 3^2\left(\frac{1}{3}\right)^2 - 3^2\left(\frac{1}{3}\right)^3 + \ldots$ $\quad$ Converges (+.5)

$\sum_{n=0}^{\infty} 3^2\left(\frac{1}{3}\right)^n(-1)^n$ geometric series? (+.5) $\quad$ to (+.5) $\dfrac{9}{1 - \left(-\frac{1}{3}\right)} = \dfrac{27}{4}$

5. (WebHW3 #9) A company buys a machine for \$575,000 that depreciates at a rate of 10% per year.

(a) [2] Find a formula for the value, V, of the machine after n years.

$575{,}000(.90)^n$ $\quad$ (+.5) start

(+.5) (+1)

after 1 year value is $575{,}000 \cdot .9$
after 2 years .9 again so $575000 \cdot .9^2$

(b) [2] Find and interpret $\lim_{n \to \infty} V(n)$.

(+1) $\lim_{n \to \infty} 575{,}000(.90)^n = 0$ $\quad$ b/c $.9 < 1$

(+1) the value of the machine will depreciate to zero