Final

TMath 126

Practice

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and will likely have a different layout.

1. TRUE/FALSE: Identify a statement as True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, identify it as false and provide a counterexample.

   Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be vectors in $\mathbb{R}^3$.
   Recall that $\cdot$ refers to the dot product, and $\times$ refers to the cross product.

   (a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges to a finite number.

   (b) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that the n^{th} partial sum of a series is $s_n = \dfrac{n+5n^2}{n^2-e}$. Then $\lim_{n\to\infty} a_n = 5$.

   (c) Newton's method will approach a root of a function, if it exists, no matter the initial guess.

(d) $||\overrightarrow{a} \times \overrightarrow{b}|| = ||\overrightarrow{b} \times \overrightarrow{a}||$.

(e) $(\overrightarrow{a} \times \overrightarrow{b}) \cdot \overrightarrow{a} = 0$.

(f) Let f be a function of x and y. If $\nabla f(c,d) = (2,1)$, then the vector $\langle 2,1 \rangle$ is tangent to the contour line of the surface of f at $(c,d,f(c,d))$.

(g) $\int_{-1}^{2} \int_{0}^{6} x^2 \sin(x-y)\,dx\,dy = \int_{0}^{6} \int_{-1}^{2} x^2 \sin(x-y)\,dy\,dx$

(h) $\int_{-1}^{x} \int_{0}^{6} x^2 \sin(x-y)\,dx\,dy = \int_{0}^{6} \int_{-1}^{x} x^2 \sin(x-y)\,dy\,dx$

2. Evaluate the following if possible.

$$\lim_{n\to\infty} a_n$$

where $a_1 = 0$ and $a_{n+1} = 2^{a_n} - 3$

$$\sum_{n=0}^{\infty} \frac{n+1}{3n+2}$$

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$$

$$\lim_{n\to\infty} \sin\left(\frac{6n\pi}{5+8n}\right)$$

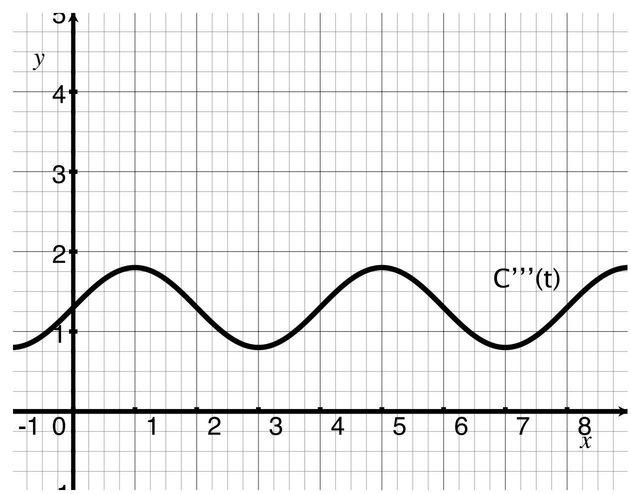
3. The temperature of a microprocessor is taken every second and only the last three readings are recorded. Below is a chart of the temperature C (in Celsius) and time t from which we estimated the first and second derivatives of C at $t = 3$.

t	2	3	4
$C(t)$	46	48	52

n	0	1	2
$C^n(3) \approx$	48	4	3

(a) Use all of the above data to estimate the values of C close to 3.

(b) Temperature changes rather slowly and experimentally we know $C^{(3)}(t)$ has the following graph. Provide an upper bound for the estimate of $C(5)$ using part (a).



4. You are given the following data of a function $g(x, y)$. Your boss wants you to approximate $g(.8, 1.4)$ and wants to be convinced you're doing something sophisticated. Find a linear approximation for your boss and explain your choices (there are many that you will make!).

x	y	$g(x, y)$
0.55	1.2	27
0.65	1.0	31
0.65	1.1	29
0.75	1.2	50

5. Let Q be the plane containing the line $L(t) = \langle 2+t, 1-t, 1-t \rangle$ and the point $(1,0,1)$. Let R be defined by $x + 2y + 3z = 0$

(a) Find an equation of a plane for Q.

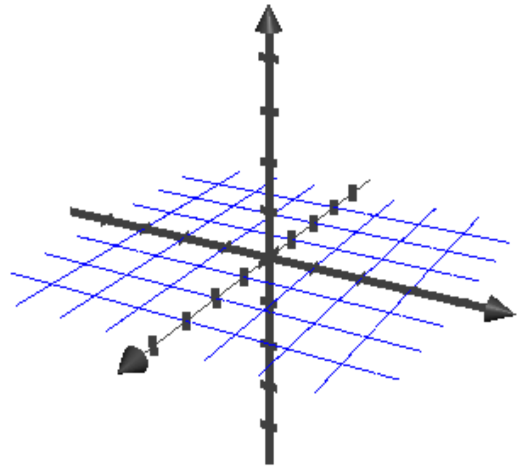
(b) Find the distance between Q and the point (2,-1,3).

(c) Identify if R is a point, line, plane, or none of the above.

(d) Given that Q and R intersect and identify the intersection as a point, line, plane, or none of the above.

(e) Find the angle that Q and R intersect.

6. Consider the vectors: $\overrightarrow{v} = \langle 1, 2, -2\rangle$ and $\overrightarrow{w} = \langle 2, -1, -2\rangle$

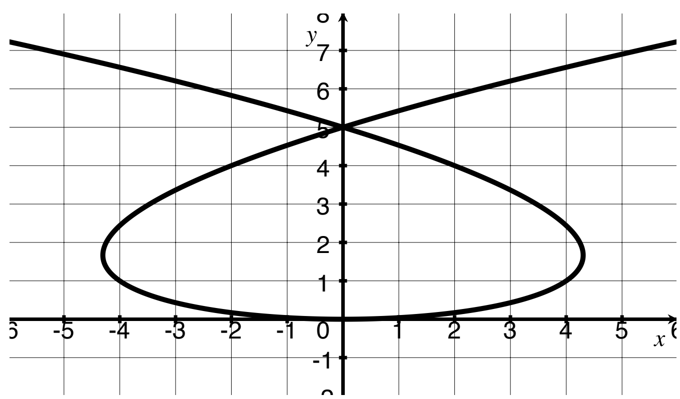


(a) Draw the vector $-\overrightarrow{v}$

(b) Draw the vector $\overrightarrow{w} - \overrightarrow{v}$

(c) Draw the vector $\overrightarrow{v} \times \overrightarrow{w}$

7. Consider the parametric equation $x(t) = t^3 - 5t$ and $y(t) = t^2$.



(a) [3] Looking at the graph, approximate where $\frac{dy}{dx}$ is not defined.

(b) [4] Find the equation of *one* of the lines tangent to the above parametric equations at $(0, 5)$.

8. Find the maximum and minimum volumes of a rectangular box with the constraints that the surface area is 1500cm^2 and total edge length is 200cm.

9. Common blood types are determined by three alleles, A, B, and O. If p is the percent of allele A in the population, q is the percent of allele b in the population and r is the percent of allele O in the population then the proportion of individuals with a mixed blood type (e.g. AB, AO or BO) is $P(p,q,r) = 2pq + 2pr + 2qr$. Find the maximal P value.

10. Consider the double integral

$$\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} \cos(x)\sqrt{1+\cos^2 x}\, dxdy$$

(a) Sketch the region in the xy-plane where the integral is taken over.

(b) Switch the order of integration.

(c) Compute the double integral.