§13.4 & 13.7 **WrittenHW #8** TMath 126

WrittenHW #8

1. Consider the volume of a right circular cylinder with radius y and height x.
   (a) [1] Write volume z, as a function of x and y.
   (b) [4] Find a linear approximation for the volume of a right circular cylinder when the radius is 2cm and the height is 3cm.
   (c) [3] It is determined that the error involved in measuring each dimension is ± 0.05cm. Use your linear approximation to provide a range of possible volumes.

2. Consider $z = 2xy^2$ and $8x^2 - 5y^2 - 8z = -13$.
   (a) [1] Verify that the two curves intersect when $x = y = 1$.
   (b) [3] Find the plane tangent to $z = 2xy^2$ when $x = y = 1$.
   (c) [3] Determine if the two surfaces are tangent to each other at the above point or if they are perpendicular. Justify your conclusions mathematically.

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1. Determine (if possible) whether $f(a,b)$ is a relative maximum, a relative minimum, a saddle point based on the following information. Justify your conclusions.

   (a) [2] $f_x(a,b) = 0$, $f_y(a,b) = 0$, $f_{xx}(a,b) = 9$, $f_{yy}(a,b) = 4$, and $f_{xy}(a,b) = f_{yx}(a,b) = 6$.

   (b) [2] $f_x(a,b) = 0$, $f_y(a,b) = 0$, $f_{xx}(a,b) = -5$, $f_{yy}(a,b) = 3$, and $f_{xy}(a,b) = f_{yx}(a,b) = 3$.

2. A company makes stationary at two locations. The cost of making x_1 units at location 1 is $C_1 = 0.02x_1^2 + 4x_1 + 500$ and the cost of producing x_2 units at the second location is $C_2 = 0.05x_2^2 + 4x_2 + 275$. The stationary is sold for \$15 per unit.

   (a) [2] Find a function that returns the profit for the company.

   (b) [3] Find the quantity that should be produced at each location to maximize the profit.

3. The Shannon diversity index S is a way to measure species diversity. If a habitat has three species A, B, and C, then its Shannon diversity index is $S = -x\ln(x) - y\ln(y) - z\ln(z)$ where x is the percent of species A, y is the percent of species B, and z is the percent of species C is in the habitat.

   (a) [2] Assume there are no other species in the habitat find an equation relating x, y, and z.

   (b) [4] Show the maximum value of S occurs when $x = y = z = \frac{1}{3}$.