§9.7 & §9.10

WrittenHW #2

TMath 126

1. Let f be a function with the following properties. Provide explanations for all of your computations!

   - $f(16) = 4$
   - $f'(16) = \frac{1}{8}$
   - $f''(16) = \frac{-1}{2^2 \cdot 4^3}$
   - $f^{(3)}(16) = \frac{3}{2^3 \cdot 4^5}$
   - $f^{(4)}(x) = \frac{-15}{2^4} x^{-\frac{7}{2}}$

   (a) [4] Use the data above to find the 3rd degree Taylor approximation of f.

   (b) [1] Approximate $f(16.5)$

2. Consider $f(x) = x^2 \cos(x)$.

   (a) [4] Find the 2nd Taylor Polynomial centered at π of f.

   (b) [1] Approximate $f\left(\frac{7\pi}{8}\right)$.

3. [5] A bottle rocket fired from the ground with initial speed v_0, follows the trajectory given by

$$y = \left(\tan(\theta) - \frac{g}{kv_0 \cos(\theta)}\right) x - \frac{g}{k^2} \ln\left(1 - \frac{kx}{v_0 \cos(\theta)}\right)$$

   where θ is the angle of projection, g is due to gravity, and k is the drag factor caused by air resistance. Computing this can take time/drain resources and generally only a quadratic approximation is needed (even for *actual* rockets!). Use the Taylor series for $\ln(1 + x)$ centered at 0 in the above and simplify terms until you have a quadratic in x.

WrittenHW #2

1. Consider again, a function f with the following properties. Provide explanations for all of your computations!

   - $f(16) = 4$
   - $f'(16) = \frac{1}{8}$
   - $f''(16) = \frac{-1}{2^2 \cdot 4^3}$
   - $f^{(3)}(16) = \frac{3}{2^3 \cdot 4^5}$
   - $f^{(4)}(x) = \frac{-15}{2^4} x^{-\frac{7}{2}}$

   (a) [2] Give an upper bound to the error when you computed $f(16.5)$.

   (b) [3] Find an upper bound to the error between $f(x)$ and the appropriate 3rd degree Taylor approximation when x is between 1 and 17.

2. [5] Find the degree of the Taylor Approximation needed so that the error in the approximation of $\ln(1.25)$ is less than .001. Be clear about what function you are using to build your Taylor Approximation!

3. [5] Show that the Taylor series centered at 0 converges to the function $f(x) = \sin(x)$ for all x. *Careful*, this is an argument, not a computation that you need to give!