§9.1 & §3.8

WrittenHW #1

TMath 126

1. [4] Find the limit of each sequence, if possible. Provide justification.

   (a) $a_n = \dfrac{n^8}{e^n}$

   (b) $\{n \cos\left(\dfrac{1}{n}\right)\}$

2. [3] Write an expression for the nth term in the sequence whose first five terms are:

$$\{-\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \frac{6}{7}, -\frac{7}{8}, ...\}$$

3. [3] Consider the recursively defined sequence where $a_n = \sin(a_{n-1}) + a_{n-1}$.

   (a) Graph the recursive function needed to use cobwebbing.

   (b) Use cobwebbing to determine all values of a_1 such that the sequence $\{a_n\}_{n=1}^{\infty}$ converges to π?

4. [5] Write down the first three numbers in the sequence resulting from using Newton's method to estimate the root on the function $f(x) = x - e^{-x}$. Provide graphical or algebraic justification/work.

§9.2 # WrittenHW #1 TMath 126

1. [6] Determine the convergence or divergence of the series. Clearly *justify* your answer!

   (a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$

   (b) $\displaystyle\sum_{n=1}^{\infty}\ln\left(\frac{1}{n}\right)$

2. [4] Many jobs offer an automatic retirement plan that moves P dollars from your paycheck into a retirement account. If the deposits are made each month for t years, and the account earns interest at the annual percentage rate r, when compounded monthly, the amount A of money in the account at the end of t years is:

$$A = P + P\left(1+\frac{r}{12}\right) + P\left(1+\frac{r}{12}\right)^2 + P\left(1+\frac{r}{12}\right)^3 + ... + P\left(1+\frac{r}{12}\right)^{12t-1}$$

   Notice that this begins like a geometric series, so we can compute the entire infinite sum $\left(\displaystyle\sum_{i=1}^{\infty} P\left(1+\frac{r}{12}\right)^i\right)$.
   Notice also that the "missing infinite terms" make another geometric series and A is the difference of these. Use these observations to find a closed form for A (a way to compute A without adding up all of the terms listed above).

3. Consider the line segment $[0,1]$ (top line in the figure below). Let s_1 be the center third of the line segment (the missing third in the second line in the figure below). Let s_2 be s_1 and the center thirds of the remaining two line segments (the missing sections in the third line in the figure below). Recursively add to s_n by taking the middle thirds of all remaining line segments.

   (a) [3] Find $\displaystyle\lim_{n\to\infty} s_n$.

   (b) [1] Define C as the complement of $\displaystyle\lim_{n\to\infty} s_n$. Given (a), what is the length of C?

   (c) [1] Identify two points in C. How many points are in C?