FINAL

TMath 126

Spring 2017

You may use:

- any kind of calculator that cannot access the internet and
- 3 double-sided 3×5" cards for this exam.

1. [1] (prerequisite material) Your Name:

2. [9] Identify each statement below as true if the statement is *always* True and provide a brief justification. Otherwise, identify the statement as False and provide a counterexample or brief justification.

   (a) (practice final #1) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that the n^{th} partial sum of the associated series is $s_n = \dfrac{n+n^2}{n^2-\pi}$. Then $\lim\limits_{n\to\infty} a_n = 0$.

   (b) (exam2 #1) If $\vec{w}(t)$ and $\vec{v}(t)$ are vector-valued functions, then $(\vec{v}\,'(3) \cdot \vec{w}\,''(3)) + \vec{v}(2)$ returns a vector valued function.

   (c) (quiz4 #1) If f is a function such that $f_x(4,-2) = 0$ and $f_y(4,-2) = 0$ then $f(4,-2)$ is a maximum or a miniumum.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

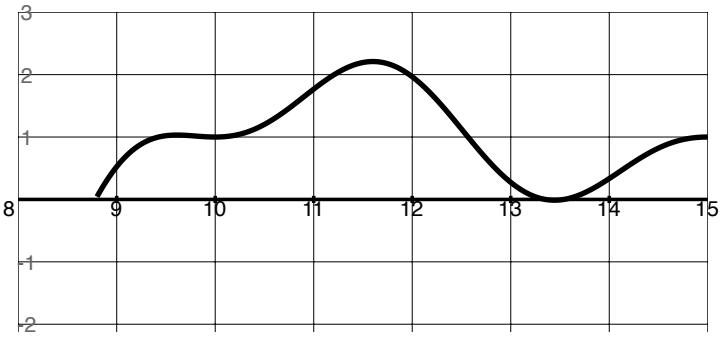
3. The number of people in line is taken every hour but only the last five readings are recorded. Below is a chart of the data N (number of people) at t o'clock. From this we can estimated the first and second derivatives of N at $t = 11$.

t	9	10	11	12	13
$N(t)$	10	7	8	22	12

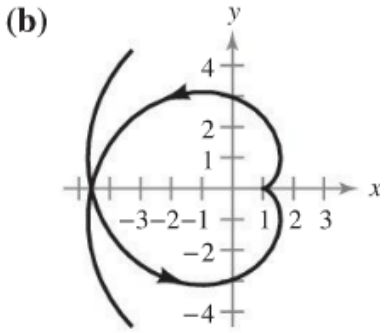
n	0	1	2
$N^n(11) \approx$	8	14	13

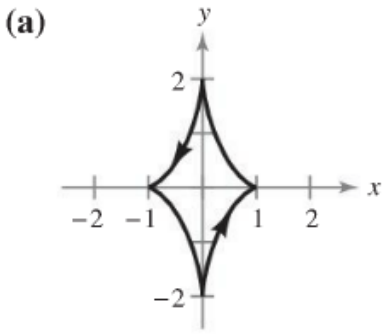
(a) [5] (HW2 §9.7 #1) Use the above approximate derivatives and data to estimate for the number of people in line at time t where t is close to 11.

(b) [3] (WebHW5 #1) The length of the line changes rather slowly & experimentally we know $N^{(3)}(t)$ has the following graph. Provide an upper bound for the above estimate of $N(14)$.



4. [4] (HW5 §10.2 #1) Match the set of parametric equations with their graph. *Justify* yourself.



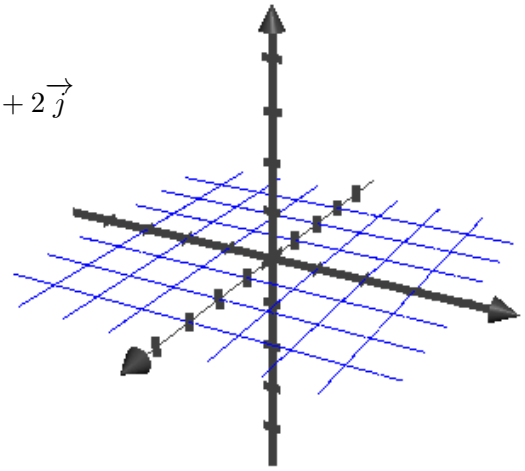


i) $x(t) = \cos^3(t)$ and $y(t) = 2\sin^3(t)$ ii) $x(t) = \cos(t) + t\sin(t)$ and $y(t) = \sin(t) - t\cos(t)$.

5. Consider the points $A(0, 0, 4)$, $B(2, 3, 0)$ and $C(1, -2, 1)$.

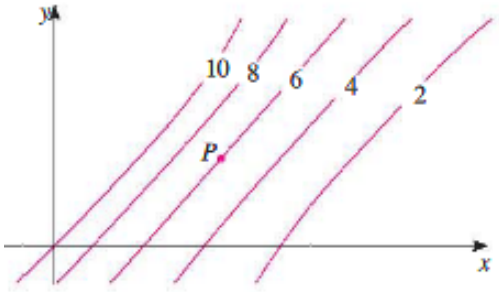
   (a) [3] (Exam2 #2) Find the components of $\overrightarrow{BA} + 2\overrightarrow{j}$

   (b) [4] (Exam2 #2) Find the angle $\angle ABC$.



   (c) [4] (WebHW8 #6) Find the equation of the plane that passes through A, B, and C.

6. Let f have the contour lines shown on the right.



(a) [3] Determine if f_y at point P is positive, negative, or zero.
Justify your answer.

(b) [2] Sketch direction of the vector $\nabla f(P)$ on the graph.

7. Let $\vec{r}(t) = \langle t + 4, t^3 - 3t \rangle$.

(a) [4] (Suggested §10.3 #25) Find the equation of the line tangent to the graph of $\vec{R}(t)$ when $x = 2$

(b) [3] (Exam2 #3) Find $\int \vec{r}(t)\, dt$.

8. [9] (5/18 Lecture) Choose *ONE* of the following. Clearly identify which of the two (a or b) you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.
For either situation a xor situation b, you need to:

   - identify the function you need to optimize and
   - outline the steps needed to find the minimums and verify this is the minimum. Make sure to highlight any steps involving calculus or precalculus techniques. You do not need to perform the steps but you do need to make sure that your process works.

   BONUS: (up to 3 points) if you outline a second method to find the minimum!

   (a) A home improvement contractor is painting the walls and ceiling of a rectangular room. The volume of the room is 998.25 ft^3. The cost of paint is \$0.12 per ft^2 and the ceiling paint cost is \$0.18 per ft^2. Find the room dimensions that will minimize costs.

   (b) Find three numbers whose sum is 42 but whose sum of squares is minimized.

9. Consider $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_{-1}^{\sin(x)} f(x,y)\,dy\,dx$

   (a) [3] (Integrated Integration Wks #3) Sketch the region of integration in the xy plane.

   (b) [3] (WebHW17 #10) Switch the order of integration.

10. [5] (Suggested §14.2 #13) Let R be the trapezoid bounded by $y = x$, $y = 2x$, $x = 1$, and $x = 2$. Find $\displaystyle\int_R \int \frac{y}{x^2 + y^2}\,dA$