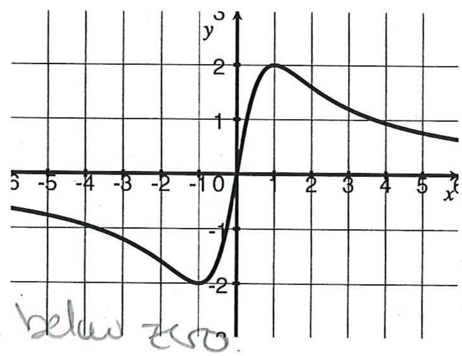
Key

Exam 2 TMath 126 /45 Spring 2017

1. [12] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample or brief justification.

§10.2
start (1.5)
answer (+.5)
true (+.5)
justify (1)

(a) The graph on the right has the parametric equation $x(t) = t - \sin(t)$ and $y(t) = 1 - \cos(t)$.



False

parametric curve (+.5)

Notice that $-1 \le \cos(t) \le 1$

So $0 \le 1 - \cos(t) \le 2$

So $0 \le y \le 2$

but on the graph y dips below zero.

§11.3, 11.4

(b) If $\vec{w}$ and $\vec{v}$ are vectors, then $(\vec{v} \cdot \vec{w}) + \vec{v}$ returns a vector.

False

vector dot or add (+.5)

$\vec{v} \cdot \vec{w}$ returns a scalar but we have no way of adding scalars $(\vec{v} \cdot \vec{w})$ and vectors $(\vec{v})$

§11.5

(c) The planes defined by $3x - y + 2z = 6$ and $0 = \langle 6, 2, 4\rangle \cdot ((x, y, z) - (0, 0, 1))$ are parallel to each other.

False

$3x - y + 2z = 6$

$3x - y + (2z - 6) = 0$

$\langle 3, -1, 2\rangle (\langle x, y, z-3\rangle) = 0$

$\iff \langle 3, -1, 2\rangle \circ (\langle x, y, z\rangle - \langle 0, 0, 3\rangle) = 0$

Note $\langle 3, -1, 2\rangle$ is not a scalar multiple of $\langle 6, 2, 4\rangle$ so the normal vectors are not parallel $\Rightarrow$ the planes are not parallel.

§11.3 + 12.1

(d) If $\vec{w}(t)$ and $\vec{v}(t)$ are vector-valued functions, then $(\vec{v}'(3) \cdot \vec{w}''(3)) + \vec{v}(2)$ returns a vector valued function.

False

vector-valued functions (+.5)

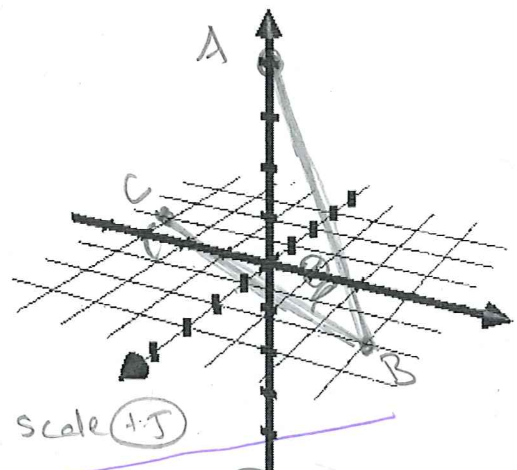
$\vec{v}'(3) \cdot \vec{w}''(3)$ is a scalar

$\vec{v}(2)$ is a vector

As highlighted in (b) we cannot add them.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider the points $A(0,0,4)$, $B(2,3,0)$ and $C(1,-2,1)$.



scale (+.5)

(a) [1] Find the components of $\overrightarrow{BA}$.

§11.2

terminal pt − initial pt

$(0,0,4) - (2,3,0)$

$\langle -2,-3,4 \rangle$

(b) [2] Find the components of $\overrightarrow{BA} + 2\overrightarrow{j}$

§11.2

$\langle -2,-3,4 \rangle + 2\langle 0,1,0 \rangle$ (+.5)

$\langle -2,-1,4 \rangle$ got it (+1)

(c) [4] Find the angle $\angle ABC$.

notation (+.5)

§11.3

We can find the angles between BA and BC

(+1) $\overrightarrow{BA} = \langle -2,-3,4 \rangle$ and $\overrightarrow{BC} = \langle -1,-5,1 \rangle$ w/ dot product

(+1) $\dfrac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\|\overrightarrow{BA}\| \, \|\overrightarrow{BC}\|} = \cos\theta$

(+.5) $\dfrac{2+15+4}{\sqrt{4+9+16}\sqrt{1+25+1}} = \cos\theta$ (+.5)

$\to \dfrac{21}{\sqrt{29}\sqrt{27}} = \cos\theta$

$\theta = \arccos\left(\dfrac{21}{\sqrt{29}\sqrt{27}}\right)$ solve (+.5)

$\approx 41°$

3. Let $\overrightarrow{r}(t) = \langle t\sin t, 3\sqrt{t}, e^t \rangle$

(a) [3] Find $\overrightarrow{r}'(t)$.

§12.2 #19+47

$\vec{r}'(t) = \langle t\cos t + \sin t, \; 3\tfrac{1}{2}t^{-1/2}, \; e^t \rangle$

(+1) (+1) (+1)

(b) [3] Find $\int \overrightarrow{r}(t)\,dt$

$\int t\sin t\,dt = -t\cos t - \int -\cos t\,dt = -t\cos t + (\sin t) + c_1$

$\int u\,dv = uv - \int v\,du$

Check: $[-t\cos t + \sin t + c_1]'$

$+t\sin t - \cancel{1\cos t} + \cancel{\cos t} + 0$ ✓

$u = t$ $du = dt$

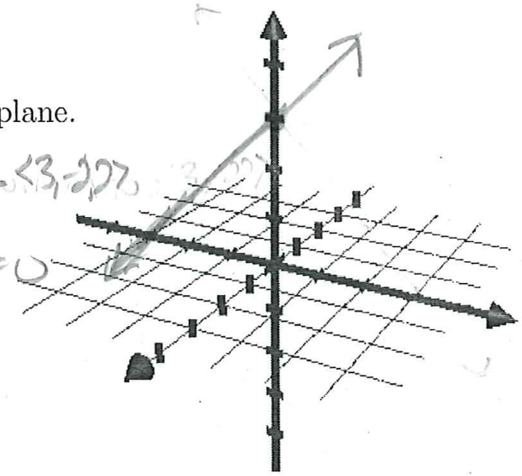
$dv = \sin t\,dt$ $v = -\cos t$

$\int \vec{r}(t)\,dt = \langle -t\cos t + \sin t + c_1, \; 2t^{3/2} + c_2, \; e^t + c_3 \rangle$

(+1) (+1) (+1)

4. Consider M defined by all (x, y, z) such that $3x - 2y + 2z = 6$.

$3x - 2y + 2(z-3) = 0$



(a) [2] Identify M as a point, vector, line, or plane. Justify your answer.

HW4 Section 11.5 #1

(+1) A plane b/c (+1) it is all vectors $\perp$ to $\langle 3,-2,2\rangle$

$\langle 3,-2,2\rangle \cdot (\langle x,y,z\rangle - \langle 0,0,3\rangle) = 0$

(b) [1] What kind of object results when M intersects the yz plane?

Lines & Planes WKS #1

A line

(c) [2] Find where M intersects the yz plane.

Lines + Planes WKS #1

when $x=0$ (+1)

So $3\cdot 0 - 2y + 2z = 6$

$\Rightarrow 2z = 2y + 6$

$\Rightarrow z = y + 3$

got one (n)

So $x=0$ and $z = y+3$ (graphed above)

– or –

$\langle 0,1,1\rangle t + (0,0,3) = (x,y,z)$ where $t \in \mathbb{R}$

– or –

Written HW4 § 11.4

5. [5] Consider a .28 meter wrench is turning a bolt shown below. If the force acting on the wrench is 92 Newton meters, what is the torque experienced by the bolt?

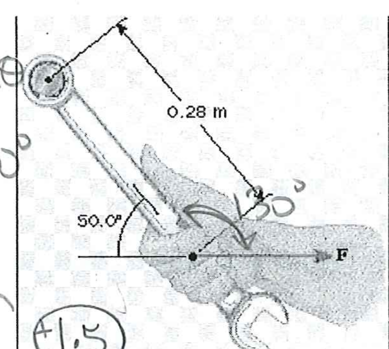
(+1)

(+1) $\overrightarrow{\text{Torque}} = \vec{r} \times \vec{F}$ or $\|\overrightarrow{\text{Torque}}\| = \|\vec{r}\|\|\vec{F}\| \sin\theta$

(+.5) (+.5) (+.5)

$= .28 \cdot 92 \cdot \sin 130°$

$= 19.73$ (+.5)



Then the direction is into the page (+1.5) (using the right hand rule)

vector/right hand (+.5)

axis (+.5) $\vec{j}$, $\vec{i}$

$\vec{F} = \langle 92, 0, 0\rangle$ (+.5)

(+1) $\vec{r} = \langle -.28\cos 50°, .28\sin 50°, 0\rangle$

50°

Sohcahtoa (+.5)

$$\vec{T} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -.28\cos 50° & .28\sin 50° & 0 \\ 92 & 0 & 0 \end{vmatrix}$$

(+1)

$\vec{T} = (0-0)\vec{i} - (0-0)\vec{j} + (0 - 92 \cdot .28\sin 50°)\vec{k}$

$= -25.76 \sin 50° \vec{k}$

$= -19.7\vec{k}$ out of the page

or 19.7 into the page

got it (+.5)

§10.2

6. [3] Consider the parametric equations $x(t) = t-3$ and $y(t) = \frac{t}{t-3}$. Write the corresponding rectangular equation by eliminating the parameter and then graph the function.

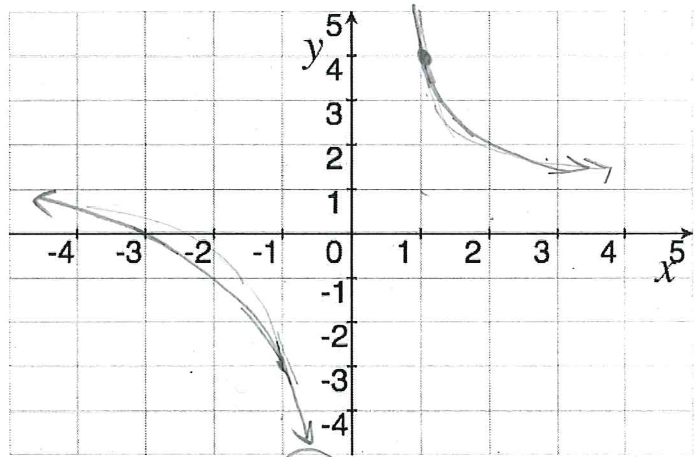
$x = t-3$

$x+3 = t$

(subbing) (+1)

$y = \frac{x+3}{x}$ (+.5) (+.5) or $\frac{x}{x} + \frac{3}{x} = \frac{3}{x} + 1$

so graph of $y = 1/x$ vertically stretched by 3 and shifted up by 1



graph (+1)

§10.3 #25 and wks

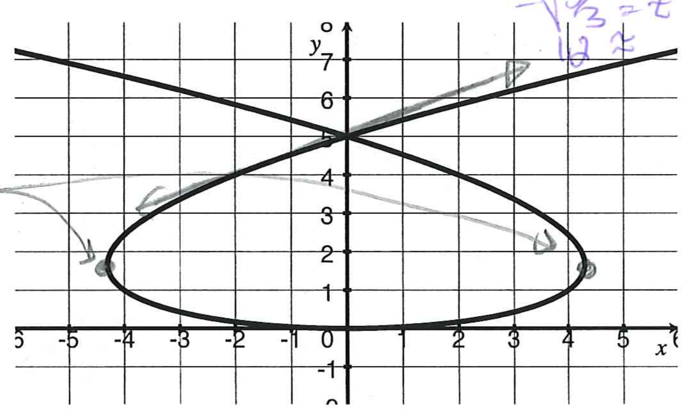
7. Consider the parametric equation $x(t) = t^3 - 5t$ and $y(t) = t^2$.

$0 = 3t^2 - 5$

$\pm\sqrt{\frac{5}{3}} = t$

$1.2 \approx t$

(a) [3] Looking at the graph, approximate where $\frac{dy}{dx}$ is not defined.



(+1) [at cusps or where there is a vertical tangent line so

$\approx (-4.4, 1.6)$ and $(4.4, 1.6)$

(+1) (+1)

(b) [4] Find the equation of *one* of the lines tangent to the above parametric equations at $(0, 5)$.

(+.5) Looking for $y = mx + b$

m: slope of line tangent to graph @ $(0,5)$

$= \left.\frac{dy}{dx}\right|_{(0,5)}$ (+.5)

$= \frac{\sqrt{5}}{5}$

plug in value to get numerical value (+1)

plug in pt. (+.5) $y - 5 = \frac{\sqrt{5}}{5}(x - 0)$

$.447$

[finding dy/dx:

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2-5}$ (+.5) (+.5) (+.5)

[finding $\left.\frac{dy}{dx}\right|_{(0,5)}$

$5 = y = t^2 \Rightarrow t = \pm\sqrt{5}$

note $(\pm\sqrt{5})^3 - 5(\pm\sqrt{5}) = 0$ ✓

So $\left.\frac{dy}{dx}\right|_{(0,5)} = \frac{2\sqrt{5}}{3\cdot 5 - 5} = \frac{2\sqrt{5}}{10} = \frac{\sqrt{5}}{5}$