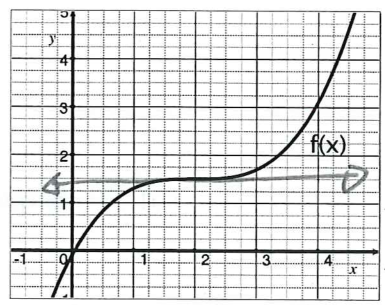
Key
None (+1)

Exam 1 TMath 126 45 Spring 2017

1. [12] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide a counterexample or brief justification.

   (a) (Suggested §3.8 #23)) Newton's method will find the root of f (graphed on the right) with a sequence starting at $x = 2$.



start (+.5)
answer (+.5)
true (+.5)
justify (+1)

False:
@ $x=2$ we can see $f'(2)=0$.
The tangent line is thus horizontal
to the x-axis and will never intersect
the x-axis meaning we can't get a second approx/term in seq.

Know Newton's method (+.5)

   (b) (VectorWks #2) The length of a vector $\vec{v}$ with components $\langle 1, 3, -2\rangle$ is $\sqrt{2}$.

False: length is $\underbrace{\sqrt{1^2+3^2+(-2)^2}}_{(+.5)\text{ formula}} = \sqrt{1+9+4} = \sqrt{14}$

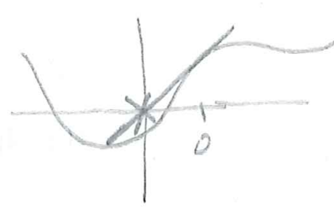
   (c) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(+.5)

False: A counterexample is the harmonic series $\sum_{n=1}^{\infty} 1/n$
Note that $\lim_{n\to\infty} 1/n = 0$ but we know $\sum_{n=1}^{\infty} 1/n$ diverges

   (d) The first degree Taylor polynomial of a function f centered at 2 is the same as the line tangent to f when x=2.

True:



The first deg. Taylor polynomial is
the polynomial $T_1(x) = mx+b$
with $T_1(2) = f(2)$ and $T_1'(2) = m = f'(2)$.
So $T_1(2)$ has the same slope as line tangent to f at x=2
AND touches f when x=2. They have to be
the same line.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Consider: $\left\{ \frac{1}{1}, \frac{-1}{1}, \frac{1}{2}, \frac{-1}{6}, \frac{1}{24}, \frac{-1}{120}, \ldots \right\}$

(a) [1] Identify the above as a series or a sequence. Explain why.

Sequence (+.5) b/c it is a list (+1) and not a list of sums

(b) [3] (SequenceWks #1) Find a formula for the general a_n term.

Start (+.5)
notation/clear (+.5)

let $a_0 = \frac{1}{1}$, $a_1 = \frac{-1}{1}$, $a_2 = \frac{1}{2}$, $a_3 = \frac{-1}{6}$, etc

then $a_n = (-1)^n \frac{1}{n!}$ where $0!$ is defined as 1

(+1) (+1)

(c) [3] (Sumer14 Exam1 #2) Determine ~~what~~ if the above converges or diverges. Justify your work.

Start (+.5)
notation/clear (+.5)

Converges. (+1)

Note if $a_n = (-1)^n \frac{1}{n!}$
(+1) The limit of the denominator is ∞.
Since "$\frac{1}{\text{large}}$ = small" the total limit is zero.

3. Compute the following if possible.

(a) [4] (WebHW3 #) $\sum_{n=1}^{\infty} \arctan(\pi n)$. (+1) Note $\lim_{n\to\infty} \arctan(\pi n) = \frac{\pi}{2}$

Start (+.5)
notation/clear (+.5)

So we are repeatedly adding (relatively) large #'s to our infinite sum. The series will diverge (+1) to infinity.
connect to series (+1)

More room

(b) [4] (HW1 §9.2 #3) The series $\frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^n$.

(+.5) (+.5)

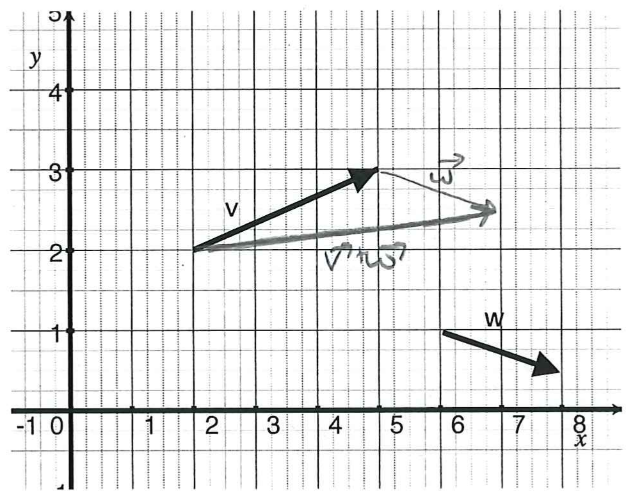
$\frac{1}{3} + \left[\frac{1}{3}\left(\frac{2}{3}\right) + \frac{1}{3}\left(\frac{2}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}\right)^3 + \ldots \right]$ geometric series?

Start (+.5)
notation/clear (+.5)

$\frac{1}{3} + \frac{2/9}{1 - 2/3} = \frac{1}{3} + \frac{2/9}{1/3} = \frac{1}{3} + \frac{2}{3} = 1$

(+.5) (+.5) (+.5) (+.5)

4. Let $\vec{v}$ and $\vec{w}$ be shown below.



(a) [2] (Suggested §11.1 #1) Find the components of $\vec{v}$.

looking @ $\vec{v}$ (+.5)
notation (+.5)

$\langle 3, 1 \rangle$
(+.5) (+.5)

(b) [1] (Suggested §11.2 #51) Identify the initial point of $\vec{v}$ shown to the right.

notation (+.5)

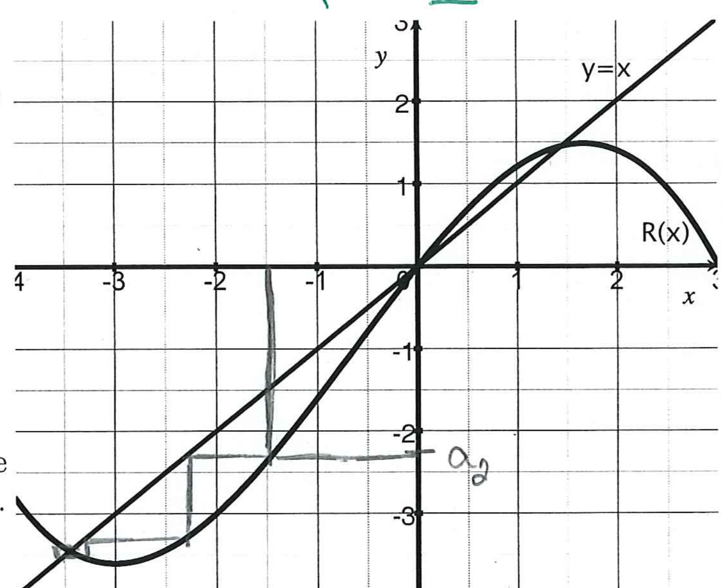
$(2,2)$
(+.5)

(c) [2] (VectorWks #2) Sketch the vector $\vec{v} + \vec{w}$ on the graph above. Be sure to *label* which vector is $\vec{v} + \vec{w}$!

moved $\vec{w}$ or $\vec{v}$ (+1)
connected correct ends (+1)

Note components $\langle 5, \frac{1}{2} \rangle$
partial +1

5. (Quiz1 #2) The graph of $R(x)$ and $y = x$ are both graphed to the right. Consider the recursively defined sequence where $a_n = R(a_{n-1})$ and $a_1 = -1.5$.



(a) [1] (SequenceWks #1) Use the graph to estimate a_2.

-2.3

(b) [2] (WrittenHW1§9.1 #3) Use the graph to estimate $\lim_{n\to\infty} a_n$.

≈ -3.5
(+1)

webbing (+1)

6. The temperature of a microprocessor is taken every second and only the last three readings are recorded. Below is a chart of the temperature C (in Celsius) and time t from which we estimated the first and second derivatives of C at $t = 3$.

t	2	3	4
$C(t)$	~~50~~ 46	~~52~~ 48	~~56~~ 52

n	0	1	2
$C^n(3) \approx$	~~52~~ 48	4	3

5 (a) ~~[4]~~ (HW2 §9.7 #1) Use all of the above data to estimate for $C(6)$.

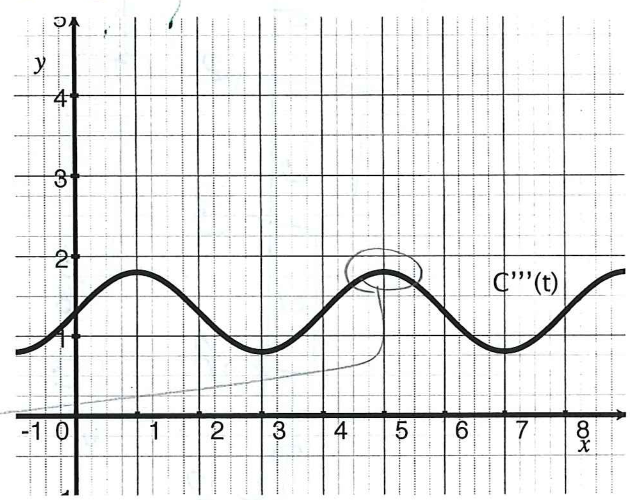
(+1) We can use 2nd degree Taylor Approx? We'll need to center it at 3. (+1) b/c that's data we have

$C(x) \approx C(3) + \frac{1}{1!}(x-3)^1 C'(3) + \frac{1}{2!}(x-3)^2 C''(3)$ form (+1)

$C(x) \approx 48 + 4(x-3) + \frac{3}{2}(x-3)^2$ plug in $C^{(n)}(3)$ (+1)

$C(5) \approx 48 + 4(5-3) + \frac{3}{2}(5-3)^2 \approx 62$ put 5 in (+1)

(b) [3] (WebHW5 #1) Temperature changes rather slowly and experimentally we know $C^{(3)}(t)$ has the following graph. Provide an upper bound for the error you found in part (a). form (+1)



$\text{Error}(x) \le \frac{1}{3!}(x-3)^3 \max|C^{(3)}(z)|$ (+.5)

where z is between 3 + 7

max @ $x = 5$ w/ value < 2 (+1)

Error (+.5)

$\text{Error}(5) \le \frac{1}{3!}(5-3)^3 \cdot 2 = 2.67$

~~(c)~~ [3] (HW2 Error #2) Microprocessors should not operate above 60° C for extended periods of time. Use the approximation in (a) to determine when the fan needs to be turned on to cool the microprocessor.