TMATH 126: Quiz 5

Key

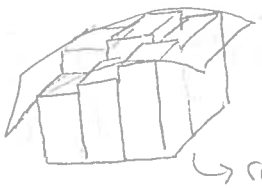
You may use any work of yours that you made from last week. This includes, practice problems from the book and worked out WebAssign problems. This *does not* include photocopies of notes from the book or tutorials shown on WebAssign. You may also use a calculator, but you are not allowed to use any device that can access the internet.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, circle F and provide either counterexample or reasoning for your answer.

T (F) Riemann sums have nothing to do with double integrals.

Double integrals are the limit of the Riemann sum as we let the # of approximating boxes go to ∞.



↳ region under the curve $z=f(x,y)$ with 6 approx boxes

Stated +.5
set-up +.5
reason +.5
clear +.5

(T) F $\displaystyle\int_0^2\int_1^3 (20x^3-36x^2y^2)\,dy\,dx = \int_1^3\int_0^2 (20x^3-36x^2y^2)\,dx\,dy.$

True Fubini's theorem

T (F) $\displaystyle\int_{-2}^0\int_x^{\sqrt{x+2}} (20x^3-36x^2y^2)\,dy\,dx = \int_x^{\sqrt{x+2}}\int_{-2}^0 (20x^3-36x^2y^2)\,dx\,dy.$

False if we switch the order of integration we don't just switch the elongated $\int$'s but need to describe the same regions with functions of y (instead of x).

2. [4] Calculate the double integral: $\iint_R 2\cos(x+2y)\,dA$ over the region $R = \{(x,y) | 0 \le x \le 3\pi, 0 \le y \le \frac{\pi}{2}\}$

$\int_0^{\pi/2}\int_0^{3\pi} 2\cos(x+2y)\,dx\,dy = \int_0^{\pi/2} 2\sin(x+2y)\Big]_0^{3\pi} dy$ (int once +.5)

(+1) limits agree w/ dx
(+1) limits on ends

$= \int_0^{\pi/2} 2\sin(3\pi+2y) - 2\sin(2y)\,dy = -\frac{1}{2}2\cos(3\pi+2y) + \frac{1}{2}2\cos(2y)\Big]_0^{\pi/2}$ (FTC +.5) (int twice +.5)

$= -\cos(3\pi+\pi) + \cos(\pi) - [-\cos(3\pi) + \cos 0] = -1 + -1 - (+1+1)$ (FTC +.5)

$= -2 - 2 = -4$

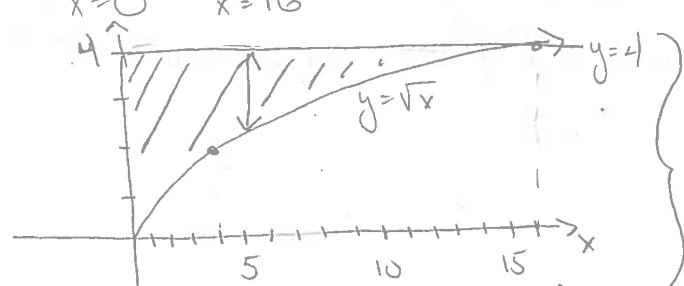
3. [3] Reverse the order of integration in the following *but do not integrate*

$$\int_0^{16}\int_{\sqrt{x}}^{4} \frac{5}{y^3+1}\,dy\,dx$$

started (+.5)

$y=4 \quad y=\sqrt{x}$

$x=0 \quad x=16$



picture +1 partial

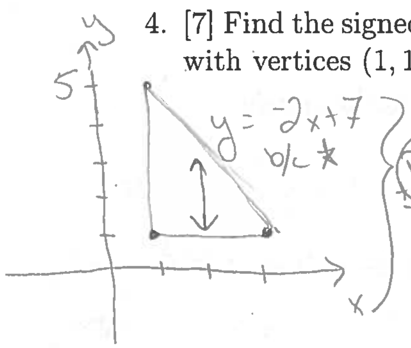
$\Rightarrow y^2 = x$

(+1) (+1) (+.5)

$\int_0^4\int_0^{y^2} \frac{5}{y^3+1}\,dx\,dy$

$4x^2 - 28x + 49$

4. [7] Find the signed volume of the solid under the surface $z = 6xy$ and above the triangle with vertices $(1,1)$, $(3,1)$, and $(1,5)$.



picture +1

$\int_1^3\int_1^{-2x+7} 6xy\,dy\,dx = \int_1^3 3xy^2\Big]_1^{-2x+7} dx$ (+1) (+.5) (+1) (+1) (+1) (int once +.5)

note eq of line
$y = \frac{-4}{2}x + b$
passes thru $(3,1)$
$1 = -2\cdot 3 + b$
$\rightsquigarrow b = 7$

$= \int_1^3 3x(-2x+7)^2 - 3x\cdot 1\,dx = \int_1^3 12x^3 - 84x^2 + 147x - 3x\,dx$ (FTC +.5)

$= \int_1^3 12x^3 - 84x^2 + 144x\,dx = 3x^4 - 28x^3 + 72x^2\Big]_1^3$ (int twice +.5)

$= (3\cdot 81 - 28\cdot 27 + 72\cdot 9) - (3 - 28 + 72) = 88$